International Asset Pricing with Recursive Preferences

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Motivation

We would like to explain:

1. The forward premium anomaly: the tendency of high interest rate currencies to appreciate

2. The Backus and Smith anomaly: the lack of correlation between consumption differentials and FX movements
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A general equilibrium model: quantities (consumption, NX,...) and prices (assets’ returns, FX,...) are outcome of utility maximization problem
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A general equilibrium model: quantities (consumption, NX,...) and prices (assets’ returns, FX,...) are outcome of utility maximization problem

The model should be consistent with:
- low int’l correlation of consumption and output
- smoothness of exchange rates
- large int’l equity risk premia
- large int’l correlation of returns
- volatility of Net Exports
- ...

Anomalies pre- and post-1970

**UIP Regression Coefficient**

- 1960: 1
- 1970: 0.6
- 1980: 0.4
- 1990: 0.2
- 2000: 0

**Correlations of Consumption Growth and FX**

- 1960: -0.2
- 1970: 0
- 1980: 0.2
- 1990: 0.4
- 2000: 0.6

**|Current Account|GDP (US−UK average)**

- 1950: -1.5
- 1960: -1
- 1970: -0.5
- 1980: 0
- 1990: 0.5
- 2000: 1

**Volatility of FX growth rate**

- 1960: 1
- 1970: 2
- 1980: 3
- 1990: 4
- 2000: 5
Anomalies pre- and post-1970

UIP Regression Coefficient

Correlations of Consumption Growth and FX

|Current Account|/GDP (US–UK average)

Volatility of FX growth rate
Our explanation

Capital mobility

- pre-1970 → financial autarky
- post-1970 → complete markets
Our explanation

1. Capital mobility
   - pre-1970 $\rightarrow$ financial autarky
   - post-1970 $\rightarrow$ complete markets

2. Recursive risk-sharing
   - agents have Epstein-Zin preferences
   - agents consume a bundle of domestic and foreign goods
   - exposure to both and short- and long-run risks
Preferences

- Two countries: home ($h$) and foreign ($f$)
- Agents have Epstein and Zin preferences

\[
U_{i,t} = (1 - \delta) \frac{C_{i,t}^{1 - \frac{1}{\psi}}}{1 - \frac{1}{\psi}} + \delta E_t \left[ U_{i,t+1}^{1-\theta} \right]^{\frac{1}{1-\theta}}, \quad \forall i \in \{h, f\}
\]

where \( \theta = \frac{\gamma - 1/\psi}{1 - 1/\psi} \).
Preferences

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\[
U_{i,t} \approx (1 - \delta) \frac{C_{i,t}^{1 - \frac{1}{\psi}}}{1 - \frac{1}{\psi}} + \delta E_t[U_{i,t+1}] + \frac{\delta}{2} \kappa_t \left( \frac{1}{\psi} - \gamma \right) V_t[U_{i,t+1}]
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**Note:**

→ When \(\gamma = \frac{1}{\psi}\): back to Expected Utility.
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- When \(\gamma = \frac{1}{\psi}\): back to Expected Utility.
- When \(\gamma > \frac{1}{\psi}\): Conditional Wealth Risk matters!
Utility Mean-Variance Trade-off
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Conditional Expected Utility vs. Conditional Volatility of Utility

- Expected Utility (no trade-off)
- \( \psi = 0.67 \)
- \( \psi = 1.5 \)
- \( \psi = 1.0 \)
Utility Mean-Variance Trade-off

- Conditional Expected Utility
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Utility Mean-Variance Trade-off

ψ = 1.5
ψ = 1.0
ψ = 0.67

Expected Utility
(no trade-off)
Total Amount of Risk

The total amount of risk decreases in \( \psi \).
Preferences

- Two countries: home (h) and foreign (f)

- Agents have Epstein and Zin preferences

\[ U_{i,t} \approx (1 - \delta) \frac{C_{i,t}^{1 - \frac{1}{\psi}}}{1 - \frac{1}{\psi}} + \delta E_t[U_{i,t+1}] + \frac{\delta}{2} \kappa_t \left( \frac{1}{\psi} - \gamma \right) V_t[U_{i,t+1}] \]

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→ When \( \gamma > \frac{1}{\psi} \): Conditional Wealth Risk matters!
→ When \( V_t[U_{i,t+1}] = 0 \): back to Expected Utility.
Preferences

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→ When \(\gamma > \frac{1}{\psi}\): Conditional Wealth Risk matters!

→ When \(V_t[U_{i,t+1}] = 0\): back to Expected Utility.

- Preferences are defined over consumption aggregates

\[
C_{h,t} = (x_{h,t})^\alpha (y_{h,t})^{1-\alpha} \quad \text{and} \quad C_{f,t} = (x_{f,t})^{1-\alpha} (y_{f,t})^\alpha, \quad \alpha > 1/2.
\]
Endowments

- Endowments' growth is *almost* i.i.d.

\[
\begin{align*}
\Delta \log X_t &= \mu_x + z_{1,t-1} + \varepsilon_{x,t} - \tau \log(X_t/Y_t) \\
\Delta \log Y_t &= \mu_y + z_{2,t-1} + \varepsilon_{y,t} + \tau \log(X_t/Y_t)
\end{align*}
\]

where \( z_{1,t} \) and \( z_{2,t} \) are small, predictable components

\[
\begin{align*}
z_{1,t} &= \rho_1 z_{1,t-1} + \varepsilon_{1,t} \\
z_{2,t} &= \rho_2 z_{2,t-1} + \varepsilon_{2,t}
\end{align*}
\]

- Shocks are *homoskedastic*.
Markets

- *Home* (*Foreign*) is endowed with good $X$ ($Y$);

- Complete Markets:

  \[
  x_{h,t} + p_t y_{h,t} + \sum_{s_{t+1}} Q_{t+1}(s_{t+1}) A_{t+1}(s_{t+1}) \leq X_t + A_t \quad [\text{Home}]
  \]

  \[
  x_{f,t} + p_t y_{f,t} - \sum_{s_{t+1}} Q_{t+1}(s_{t+1}) A_{t+1}(s_{t+1}) \leq p_t Y_t - A_t \quad [\text{Foreign}]
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  \]

- Financial Autarky: $A_t = 0.$
Results with Autarky

Only short-run (static) risk-sharing:

\[ x_{aut} = \alpha X_t, \]

\[ y_{aut} = (1 - \alpha) Y_t. \]

Exchange rate reflects short-run relative supplies:

\[ \Delta e_t = (2\alpha - 1)(\Delta x_t - \Delta y_t). \]
Results with Autarky: boring
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- Only short-run (static) risk-sharing:

\[ x_{t, aut} = \alpha X_t, \quad y_{t, aut} = (1 - \alpha) Y_t \]
Results with Autarky: boring

- Only short-run (static) risk-sharing:

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- Exchange rate reflects short-run relative supplies:

\[ \Delta e_t = (2\alpha - 1)(\Delta x_t - \Delta y_t). \]
Results with Complete Markets

Allocations:

\[ x_t^h = x_t^{h, aut} \left[ 1 + \frac{(1 - \alpha)(S_t - 1)}{1 - \alpha + \alpha S_t} \right], \quad y_t^h = y_t^{h, aut} \left[ 1 + \frac{\alpha(S_t - 1)}{\alpha + (1 - \alpha)S_t} \right] \]

where

\[ \frac{S_t}{S_{t-1}} \approx \frac{M_t^h}{M_t^f} \]
Results with Complete Markets

- Allocations:
  \[
  x^h_t = x^{h,\text{aut}}_t \left[ 1 + \frac{(1 - \alpha)(S_t - 1)}{1 - \alpha + \alpha S_t} \right], \quad y^h_t = y^{h,\text{aut}}_t \left[ 1 + \frac{\alpha(S_t - 1)}{\alpha + (1 - \alpha)S_t} \right]
  \]

- Economic interpretation:
  - \( S_t \downarrow \), when home gets good (short- or long-run) news.
  - Equivalently, countries export more in good times.
Results with Complete Markets

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- Why does this happen?

  - Agents are willing to trade-off lower consumption today for smoother future utility profiles.
  - Volatilities are high in bad times and low in good times.
Utility Mean-Variance Trade-off

\[ \psi = 0.67 \]
\[ \psi = 1.0 \]
\[ \psi = 1.5 \]

Expected Utility (no trade-off)
Results with Complete Markets

- **Allocations:**

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- **Economic interpretation:**
  - \( S_t \downarrow \), when home gets good (short- or long-run) news.
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- **Why does this happen?**
  - Agents are willing to trade-off lower consumption today for smoother future utility profiles.
  - Volatilities are high in bad times and low in good time.
Exchange rates are functions of relative supplies of the two goods
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- Both current

\[ \Delta e_t = f \left( \begin{array}{c} \varepsilon_{x,t} - \varepsilon_{y,t} \\ \geq 0 \end{array} \right) \]
Exchange rates are functions of relative supplies of the two goods

- Both current and future

\[ \Delta e_t = f \left( \varepsilon_{x,t} - \varepsilon_{y,t}, \varepsilon_{1,t} - \varepsilon_{2,t} \right) \]
Exchange rates are functions of relative supplies of the two goods

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- Agents are extremely sensitive to long-run news
Exchange rates are functions of relative supplies of the two goods

- Both current and future

\[ \Delta e_t = f \left( \frac{\varepsilon_{x,t} - \varepsilon_{y,t}, \varepsilon_{1,t} - \varepsilon_{2,t}}{>0, >0} \right) \]

- Agents are extremely sensitive to long-run news
- Long-run risks should be very correlated to replicate FX volatility
The Backus and Smith Anomaly
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The quest for $\text{corr}(\Delta c^h - \Delta c^f, \Delta e) \approx 0$
The Backus and Smith Anomaly

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→ Short-run shock to \( X \): home country is happy!
The Backus and Smith Anomaly

The quest for $corr(\Delta c^h - \Delta c^f, \Delta e) \approx 0$

→ Home increases consumption more than foreign.
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The Backus and Smith Anomaly

The quest for \( \text{corr}(\Delta c^h - \Delta c^f, \Delta e) \approx 0 \)

→ Home currency depreciates

\[ \Delta x, \Delta c^h - \Delta c^f, \Delta e \]
The Backus and Smith Anomaly

The quest for \( \text{corr}(\Delta c^h - \Delta c^f, \Delta e) \approx 0 \)

→ Home currency depreciates: \( \text{corr}(\Delta c^h - \Delta c^f, \Delta e) \) is positive
The Backus and Smith Anomaly

The quest for \( \text{corr}(\Delta c^h - \Delta c^f, \Delta e) \approx 0 \)

→ **Long-run shock to \( X \): home country is very happy!**
The Backus and Smith Anomaly

\[ \text{The quest for } \text{corr}(\Delta c^h - \Delta c^f, \Delta e) \approx 0 \]

\[ \rightarrow \text{Long-run shock to } X: \text{ home country is very happy!} \]
The Backus and Smith Anomaly

The quest for $corr(\Delta c^h - \Delta c^f, \Delta e) \approx 0$

→ Home consumption falls to restore equilibrium.
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The Backus and Smith Anomaly

The quest for $corr(\Delta c^h - \Delta c^f, \Delta e) \approx 0$

→ Home currency depreciates: $corr(\Delta c^h - \Delta c^f, \Delta e)$ is negative
The Backus and Smith Anomaly

The quest for $corr(\Delta c^h - \Delta c^f, \Delta e) \approx 0$
The Backus and Smith Anomaly

The quest for \( \text{corr}(\Delta c^h - \Delta c^f, \Delta e) \approx 0 \)
Forward Premium Anomaly
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Why do high interest rate currency have the tendency to appreciate?
Forward Premium Anomaly

Why do high interest rate currency have the tendency to appreciate?

Interest rate differential

\[ r^h_{t-1} - r^f_{t-1} \approx E_{t-1} [\Delta c^h_t - \Delta c^f_t] + \frac{1}{2} (V_{t-1} [\Delta c^f_t] - V_{t-1} [\Delta c^h_t]) \]
Forward Premium Anomaly

Why do high interest rate currency have the tendency to appreciate?

**Interest rate differential**

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\begin{align*}
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\end{align*}
\]

**Expected FX growth**

\[
\begin{align*}
  E_{t-1} \left[ \Delta e_t \right] & = E_{t-1} \left[ \Delta c^h_t - \Delta c^f_t \right] + \frac{1}{2\theta^2} \left( Var_{t-1} \left[ U^h_t \right] - Var_{t-1} \left[ U^f_t \right] \right)
\end{align*}
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Forward Premium Anomaly

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\[ E_{t-1} [\Delta e_t] = E_{t-1} [\Delta c^h_t - \Delta c^f_t] + \frac{1}{2\theta^2} (Var_{t-1} [U^h_t] - Var_{t-1} [U^f_t]) \]

→ Assume that Home has good long-run news (\(\varepsilon_{1,t} \uparrow\))
Why do high interest rate currency have the tendency to appreciate?

**Interest rate differential**

\[
r_{t-1}^h - r_{t-1}^f \approx \Delta c_{t-1}^h - \Delta c_{t-1}^f + \frac{1}{2} \left( V_{t-1} \Delta c_t^f - V_{t-1} \Delta c_t^h \right)
\]

**Expected FX growth**

\[
E_{t-1} \Delta e_t = E_{t-1} \left[ \Delta c_{t-1}^h - \Delta c_{t-1}^f \right] + \frac{1}{2 \theta^2} \left( Var_{t-1} U_t^h - Var_{t-1} U_t^f \right)
\]

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**Forward Premium Anomaly**

Why do high interest rate currency have the tendency to appreciate?

**Interest rate differential**

\[ r_{t-1}^h - r_{t-1}^f \approx \uparrow E_{t-1} [\Delta c_t^h - \Delta c_t^f] + \frac{1}{2} (V_{t-1} [\Delta c_t^f] - V_{t-1} [\Delta c_t^h]) \uparrow \]

**Expected FX growth**

\[ E_{t-1} [\Delta e_t] = E_{t-1} [\Delta c_t^h - \Delta c_t^f] + \frac{1}{2\theta^2} (\text{Var}_{t-1} [U_t^h] - \text{Var}_{t-1} [U_t^f]) \]

→ Assume that Home has good long-run news (\( \varepsilon_{1,t}^1 \uparrow \))
Forward Premium Anomaly

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**Interest rate differential**

\[ \uparrow r^h_{t-1} - r^f_{t-1} \approx \uparrow E_{t-1} [\Delta c^h_t - \Delta c^f_t] + \frac{1}{2} (V_{t-1} [\Delta c^f_t] - V_{t-1} [\Delta c^h_t]) \uparrow \]

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Forward Premium Anomaly

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**Interest rate differential**

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**Expected FX growth**

\[ E_{t-1} [\Delta e_t] = \uparrow E_{t-1} [\Delta c_t^h - \Delta c_t^f] + \frac{1}{2\theta^2} (Var_{t-1} [U_t^h] - Var_{t-1} [U_t^f]) \downarrow \]

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Forward Premium Anomaly

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**Expected FX growth**

\[ \uparrow \downarrow E_{t-1} \left[ \Delta e_t \right] = \uparrow E_{t-1} \left[ \Delta c_t^h - \Delta c_t^f \right] + \frac{1}{2\theta^2} \left( Var_{t-1} \left[ U_t^h \right] - Var_{t-1} \left[ U_t^f \right] \right) \downarrow \]

→ Assume that Home has good long-run news \((\varepsilon_{1,t} \uparrow)\)
Forward Premium Anomaly

Why do high interest rate currency have the tendency to appreciate?

**Interest rate differential**

\[ \uparrow r_{t-1}^h - r_{t-1}^f \approx \uparrow E_{t-1} [\Delta c_t^h - \Delta c_t^f] + \frac{1}{2} (V_{t-1} [\Delta c_t^f] - V_{t-1} [\Delta c_t^h] ) \uparrow \]

**Expected FX growth**

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→ Assume that Home has good long-run news (ε_{1,t} ↑)
Forward Premium Anomaly ✓

Why do high interest rate currency have the tendency to appreciate?

Interest rate differential

\[ \uparrow r_{t-1}^h - r_{t-1}^f \approx \uparrow E_{t-1} \left[ \Delta c_t^h - \Delta c_t^f \right] + \frac{1}{2} \left( V_{t-1} \left[ \Delta c_t^f \right] - V_{t-1} \left[ \Delta c_t^h \right] \right) \uparrow \]

Expected FX growth

\[ \downarrow E_{t-1} \left[ \Delta e_t \right] = \uparrow E_{t-1} \left[ \Delta c_t^h - \Delta c_t^f \right] + \frac{1}{2\theta^2} \left( \text{Var}_{t-1} \left[ U_t^h \right] - \text{Var}_{t-1} \left[ U_t^f \right] \right) \downarrow \]

→ Assume that Home has good long-run news (ε_{1,t} ↑)
### Post-1970: Complete Markets

#### Table 1: Results with Complete Markets

<table>
<thead>
<tr>
<th>Specification</th>
<th>DATA</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IES (ψ)</td>
<td></td>
<td>(with LRR)</td>
<td>(no LRR)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std (Δc) /Std (Δy)</td>
<td>0.87</td>
<td>0.96</td>
<td>0.87</td>
<td>0.99</td>
<td>0.99</td>
<td>0.93</td>
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<tr>
<td>ACF₁ (Δcₜ)</td>
<td>0.38</td>
<td>0.28</td>
<td>0.04</td>
<td>0.24</td>
<td>0.27</td>
<td>0.31</td>
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<tr>
<td>corr(Δcʰₜ, Δcᶠₜ)</td>
<td>0.55</td>
<td>0.59</td>
<td>0.82</td>
<td>0.42</td>
<td>0.48</td>
<td>0.81</td>
</tr>
<tr>
<td>E[rₙ]</td>
<td>1.15</td>
<td>0.68</td>
<td>0.00</td>
<td>1.15</td>
<td>1.54</td>
<td>9.17</td>
</tr>
<tr>
<td>Std[rₙ]</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1</td>
<td>0.67</td>
<td>1/γ</td>
</tr>
<tr>
<td>corr(rʰₙ,t, rᶠₙ,t)</td>
<td>0.64</td>
<td>0.88</td>
<td>-1.00</td>
<td>0.92</td>
<td>0.92</td>
<td>0.97</td>
</tr>
<tr>
<td>Std[M]/E[M]</td>
<td>27.83</td>
<td>12.99</td>
<td>70.51</td>
<td>87.78</td>
<td>16.46</td>
<td></td>
</tr>
<tr>
<td>Std (Δeₜ)</td>
<td>11.65</td>
<td>14.47</td>
<td>7.45</td>
<td>20.58</td>
<td>17.99</td>
<td>10.23</td>
</tr>
<tr>
<td>corr(Δcʰₜ - Δcᶠₜ, Δeₜ)</td>
<td>-0.02</td>
<td>-0.02</td>
<td>1.00</td>
<td>-0.53</td>
<td>-0.34</td>
<td>1.00</td>
</tr>
<tr>
<td>β₁ UIP</td>
<td>-0.72</td>
<td>-0.71</td>
<td>-155.69</td>
<td>-2.36</td>
<td>-1.59</td>
<td>1.01</td>
</tr>
<tr>
<td>E(rᵈₑ,t)</td>
<td>6.0</td>
<td>6.68</td>
<td>0.32</td>
<td>3.75</td>
<td>0.74</td>
<td>0.46</td>
</tr>
<tr>
<td>corr(rᵈₑ,t, Δeₜ)</td>
<td>0.05</td>
<td>0.08</td>
<td>0.12</td>
<td>-0.14</td>
<td>-0.17</td>
<td>0.06</td>
</tr>
<tr>
<td>corr(rᵈₑ,t, rᶠₓₜ)</td>
<td>-0.05</td>
<td>0.03</td>
<td>0.03</td>
<td>0.10</td>
<td>0.00</td>
<td>0.00</td>
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</table>

Notes - In panel A, we report our annual calibration for the post–Bretton Wood sample. Our data sources are described in appendix A. The IES varies across different specifications. For specification (2), we impose σₓ = 0 and ρₓᵧ = 0.35 so that the cross-country correlation of the output growth rates remains unchanged. The currency return is defined as rₓᵗ₊₁ = ∆eᵗ₊₁ + rᶠᵗ − rᶠₓᵗ. The equity excess return, rₑᵗ, refers to the following cash flow: ∆dᵢᵗ = λ ∆cᵢᵗ + ǫᵢᵗ, i ∈ {h, f}, where λ = 1.7 and ǫᵢᵗ ∼ i.i.d. N(0, 15²).

The role of long-run risk. In specification (2) we abstract away from international long-run risks and allow output growth to be driven only by short-run fluctuations. This model fails along several dimensions. First of all, in contrast to the data, the predicted β₁ UIP is negative and close in value to its empirical counterpart, thanks to our endogenous volatility channel. Furthermore, both exchange rate growth and currency returns are virtually uncorrelated with domestic equity excess returns, as in the data.
## Pre-1970: Financial Autarky

<table>
<thead>
<tr>
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<tbody>
<tr>
<td></td>
<td>DATA</td>
<td>Model (with LRR)</td>
</tr>
<tr>
<td>Std (Δc) / Std (Δy)</td>
<td>0.69</td>
<td>0.97</td>
</tr>
<tr>
<td>ACF1 (Δct)</td>
<td>0.41</td>
<td>0.39</td>
</tr>
<tr>
<td>corr(Δc^h_t, Δc^f_t)</td>
<td>0.02</td>
<td>-0.06</td>
</tr>
<tr>
<td>E[r_f]</td>
<td>0.61</td>
<td>1.75</td>
</tr>
<tr>
<td>Std[r_f]</td>
<td>1.72</td>
<td>0.85</td>
</tr>
<tr>
<td>corr(r^h_{f,t}, r^f_{f,t})</td>
<td>0.46</td>
<td>0.59</td>
</tr>
<tr>
<td>Std[M]/E[M]</td>
<td>27.54</td>
<td>14.55</td>
</tr>
<tr>
<td>Std (Δe_t)</td>
<td>5.59</td>
<td>3.53</td>
</tr>
<tr>
<td>corr(Δc^h_t − Δc^f_t, Δe_t)</td>
<td>0.47</td>
<td>1.00</td>
</tr>
<tr>
<td>β_{UIP}</td>
<td>0.94</td>
<td>1.50</td>
</tr>
</tbody>
</table>

Notes: - We adopt the calibration corresponding to specificat ion (1) in table 1 and adapt it to the pre-1970 sample as follows: (i) In the model with long-run risk, we set ρ_{12} = 0.50 and ρ_{12} = -0.50. This enables us to match the lower correlation of the long-run components in the pre-1970 sample and keep the unconditional correlation of output growth constant across model specifications. The data sources are explained in detail in appendix A. The last three columns refer to the change in moments of interest across the post-1970 and pre-1970 samples for the data, and across the models with internationally complete markets and financial autarky. We test the null of no change in Std(Δe_t), corr(Δc^h_t − Δc^f_t, Δe_t), and β_{UIP} across regimes and denote p-values smaller than 1% and 2% by ** and ***, respectively.
The Role of IES

The graphs illustrate the relationship between IES and various economic metrics:

- **Unconditional Expected Utility**
- **Unconditional Vol. of Utility**
- **Conditional Expected Utility**
- **Conditional Vol. of Utility**
- **Cross-Country Correlations**

Each graph shows how changes in IES affect these metrics across different values. The diagrams help visualize the economic implications of varying IES parameters.
## Testable Implications (post-1970)

### Contemporaneous Responses

#### A: Consumption Growth

\[
\Delta c^U_t - \Delta c^K_t = \mu_t + \beta_x e_{x,t} + \beta_y e_{y,t} + \beta_1 \epsilon_{1,t} + \beta_2 \epsilon_{2,t} + \epsilon_t
\]

<table>
<thead>
<tr>
<th></th>
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<th>pd,cy,dy</th>
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</thead>
<tbody>
<tr>
<td>F-stat</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Total R²</td>
<td>0.834</td>
<td>0.875</td>
<td>0.848</td>
</tr>
<tr>
<td>Long-run news R²</td>
<td>0.003</td>
<td>0.007</td>
<td>0.005</td>
</tr>
<tr>
<td>(\beta_1) (H₀: (\beta_1 \geq 0))</td>
<td>-0.013***</td>
<td>-0.018***</td>
<td>-0.003***</td>
</tr>
<tr>
<td>(\beta_2) (H₀: (\beta_2 \leq 0))</td>
<td>0.003*</td>
<td>0.007***</td>
<td>0.005***</td>
</tr>
<tr>
<td>(\beta_x) (H₀: (\beta_x \leq 0))</td>
<td>0.005***</td>
<td>0.012***</td>
<td>0.004***</td>
</tr>
<tr>
<td>(\beta_y) (H₀: (\beta_y \geq 0))</td>
<td>-0.009***</td>
<td>-0.008***</td>
<td>-0.011***</td>
</tr>
</tbody>
</table>

#### B: Excess Returns

\[
r^U_{ex,t} - r^K_{ex,t} = \mu_t + \beta_x e_{x,t} + \beta_y e_{y,t} + \beta_1 \epsilon_{1,t} + \beta_2 \epsilon_{2,t} + \epsilon_t
\]

<table>
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<tbody>
<tr>
<td>F-stat</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Total R²</td>
<td>0.874</td>
<td>0.455</td>
<td>0.493</td>
</tr>
<tr>
<td>Long-run news R²</td>
<td>0.786</td>
<td>0.255</td>
<td>0.356</td>
</tr>
<tr>
<td>(\beta_1) (H₀: (\beta_1 \leq 0))</td>
<td>0.606***</td>
<td>0.299***</td>
<td>0.086***</td>
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<tr>
<td>(\beta_2) (H₀: (\beta_2 \geq 0))</td>
<td>-0.262***</td>
<td>-0.274***</td>
<td>-0.173***</td>
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<tr>
<td>(\beta_x) (H₀: (\beta_x \leq 0))</td>
<td>0.004</td>
<td>-0.148</td>
<td>-0.003</td>
</tr>
<tr>
<td>(\beta_y) (H₀: (\beta_y \geq 0))</td>
<td>0.009</td>
<td>-0.034***</td>
<td>0.046</td>
</tr>
</tbody>
</table>

### Equity Risk Premia

#### C: Realized Variance

\[
(r^U_{ex,t})^2 - (r^K_{ex,t})^2 = \mu_t + \beta_x e_{x,t} + \beta_y e_{y,t} + \beta_1 \epsilon_{1,t} + \beta_2 \epsilon_{2,t} + \epsilon_t
\]

<table>
<thead>
<tr>
<th></th>
<th>pd only</th>
<th>pd,cy,dy</th>
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<tbody>
<tr>
<td>F-stat</td>
<td>0.092</td>
<td>0.201</td>
<td>0.234</td>
</tr>
<tr>
<td>Total R²</td>
<td>0.467</td>
<td>0.396</td>
<td>0.341</td>
</tr>
<tr>
<td>Long-run news R²</td>
<td>0.172</td>
<td>0.127</td>
<td>0.172</td>
</tr>
<tr>
<td>(\beta_1) (H₀: (\beta_1 \geq 0))</td>
<td>0.072</td>
<td>-0.091*</td>
<td>-0.058**</td>
</tr>
<tr>
<td>(\beta_2) (H₀: (\beta_2 \leq 0))</td>
<td>0.079**</td>
<td>0.127***</td>
<td>0.085**</td>
</tr>
<tr>
<td>(\beta_x) (H₀: (\beta_x \geq 0))</td>
<td>-0.020*</td>
<td>0.025</td>
<td>-0.002</td>
</tr>
<tr>
<td>(\beta_y) (H₀: (\beta_y \leq 0))</td>
<td>0.017**</td>
<td>0.047***</td>
<td>-0.001</td>
</tr>
</tbody>
</table>

#### D: Excess Returns (one period ahead)

\[
r^U_{ex,t+1} - r^K_{ex,t+1} = \mu_t + \beta_x e_{x,t} + \beta_y e_{y,t} + \beta_1 \epsilon_{1,t} + \beta_2 \epsilon_{2,t} + \epsilon_t
\]

<table>
<thead>
<tr>
<th></th>
<th>pd only</th>
<th>pd,cy,dy</th>
<th>All</th>
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<tbody>
<tr>
<td>F-stat</td>
<td>0.001</td>
<td>0.007</td>
<td>0.000</td>
</tr>
<tr>
<td>Total R²</td>
<td>0.169</td>
<td>0.128</td>
<td>0.174</td>
</tr>
<tr>
<td>Long-run news R²</td>
<td>0.158</td>
<td>0.027</td>
<td>0.126</td>
</tr>
<tr>
<td>(\beta_1) (H₀: (\beta_1 \leq 0))</td>
<td>-0.371***</td>
<td>-0.203**</td>
<td>-0.070***</td>
</tr>
<tr>
<td>(\beta_2) (H₀: (\beta_2 \leq 0))</td>
<td>0.172***</td>
<td>0.197***</td>
<td>0.169***</td>
</tr>
<tr>
<td>(\beta_x) (H₀: (\beta_x \geq 0))</td>
<td>-0.038***</td>
<td>0.052</td>
<td>-0.033**</td>
</tr>
<tr>
<td>(\beta_y) (H₀: (\beta_y \leq 0))</td>
<td>0.007</td>
<td>0.041***</td>
<td>-0.043</td>
</tr>
</tbody>
</table>
Concluding Remarks

A two-countries model with:

- complete markets
- two goods
- long-run risks in the endowments
- recursive preferences

1 generates

- dynamic risk-sharing scheme
- endogenously time varying second moments

2 replicates a number of international finance facts
Concluding Remarks

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   - dynamic risk-sharing scheme
   - endogenously time varying second moments

2 replicates a number of international finance facts

3 next step: “Backus Kehoe Kydland the Epstein Zin way”
Concluding Remarks

A two-countries model with:

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1. generates
   - dynamic risk-sharing scheme
   - endogenously time varying second moments

2. replicates a number of international finance facts

3. next step: “BKK the EZ way”
Concluding Remarks

A two-countries model with:

- complete markets
- two goods
- long-run risks in the endowments
- recursive preferences

1. generates
   - dynamic risk-sharing scheme
   - endogenously time varying second moments

2. replicates a number of international finance facts

3. next step: “BKK the EZ way”

Tomorrow: 10:15 am, Manchester Grand Hyatt, Elizabeth Ballroom F