Six anomalies looking for a model.
A consumption based explanation of international finance puzzles

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Motivation

A long list of international finance puzzles:

- Why is there an apparent disconnect between exchange rates and consumption?
- Why do high interest rate currencies appreciate?
- Why is risk-sharing high with prices and low with quantities?
- Can we account for the relative smoothness of exchange rate movements and stock markets' fluctuations?
- What is driving the high correlation of international prices, if fundamentals are almost uncorrelated?
- ...

Can we explain the disconnect between international prices and quantities?

Can we explain the disconnect between international prices and quantities?

This paper:

1. two slowly moving predictive components of consumption growth
2. risk-sensitive preferences
3. correlation of long-lasting and temporary shocks to consumption growth
4. stochastic volatility

Relation to the literature

- Bansal and Yaron (2004): long-run risks can explain equity premium puzzle
- Colacito and Croce (2007): highly cross-country correlated long-run risks can explain
  - volatility of real exchange rate movements
  - degree of int'l correlation of stock markets
- Colacito and Croce (2008): general equilibrium analysis with risk sensitive preferences
- Colacito, Croce, and Ghysels (in progress): estimation of long-run risks model

Plan of the talk

- The puzzles that we want to explain

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- The puzzles that we want to explain
- A look at the data

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- The economy

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- Inside the black box

Plan of the talk

- The puzzles that we want to explain
- A look at the data
- The economy
- Inside the black box
- Quantitative findings

Things we would like to explain

$$\boxed{m^*_{t+1} - m_{t+1} = \Delta e_{t+1}}$$

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Time Additive preferences

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Time Additive preferences

↓

$$\gamma\left(\Delta c_{t+1} - \Delta c^*_{t+1}\right) = \Delta e_{t+1}$$

Things we would like to explain

$$m^*_{t+1} - m_{t+1} = \Delta e_{t+1}$$

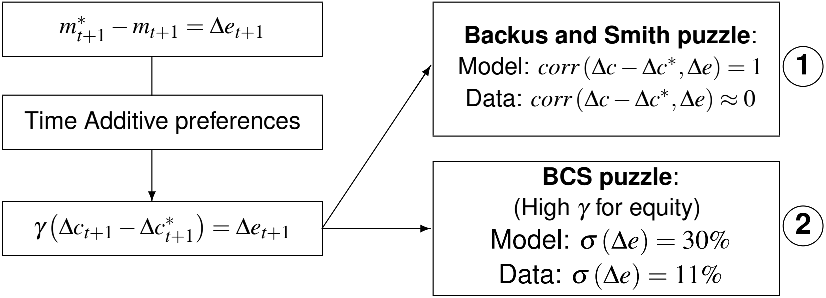
Time Additive preferences

$$\gamma\left(\Delta c_{t+1} - \Delta c^*_{t+1}\right) = \Delta e_{t+1}$$

Backus and Smith puzzle:
Model: $corr\left(\Delta c - \Delta c^*, \Delta e\right) = 1$
Data: $corr\left(\Delta c - \Delta c^*, \Delta e\right) \approx 0$

1

Things we would like to explain



Things we would like to explain

$$m^*_{t+1} - m_{t+1} = \Delta e_{t+1}$$

Time Additive preferences

$$\gamma\left(\Delta c_{t+1} - \Delta c^*_{t+1}\right) = \Delta e_{t+1}$$

Backus and Smith puzzle: ①

Model: $corr\left(\Delta c - \Delta c^*, \Delta e\right) = 1$

Data: $corr\left(\Delta c - \Delta c^*, \Delta e\right) \approx 0$

BCS puzzle: ②

(High γ for equity)

Model: $\sigma\left(\Delta e\right) = 30\%$

Data: $\sigma\left(\Delta e\right) = 11\%$

UIP anomaly: ③

$E_t[\Delta e_{t+1}] = \beta(r_t - r^*_t)$

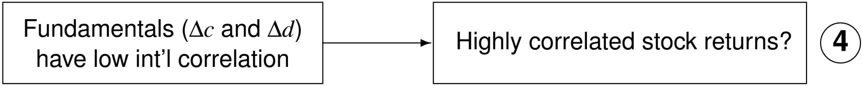
Model: $\beta = 1$

Data: $\beta << 1$

Things we would like to explain (cont'd)

Fundamentals (Δc and Δd) have low int'l correlation

Things we would like to explain (cont'd)



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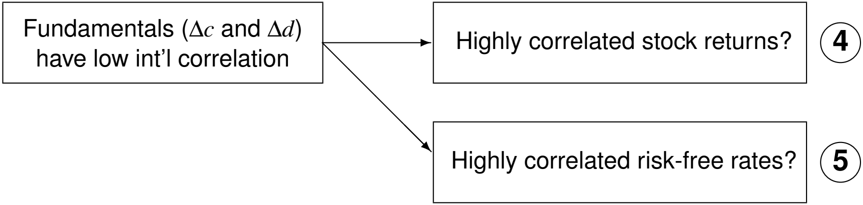

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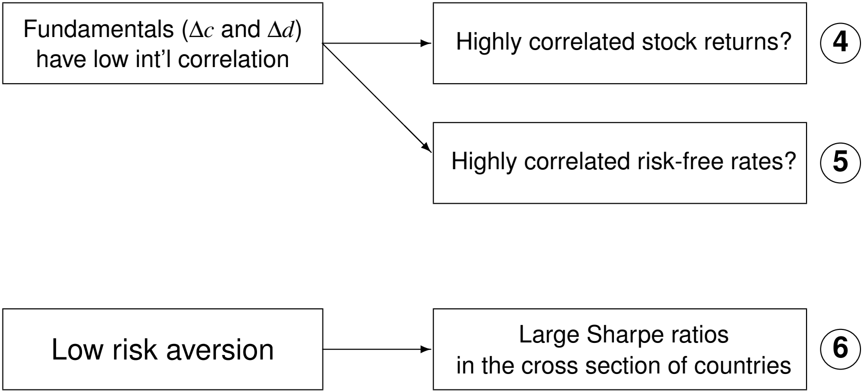

The Backus and Smith anomaly

$$corr\left(\Delta c - \Delta c^{*}, \Delta e\right)$$

	US	CAN	JPN	GER	UK	FRA	ITA	Average
US	-	-0.223	-0.013	0.166	-0.146	-0.007	-0.037	-0.043
CAN	-0.223	-	0.098	0.185	0.007	0.105	0.153	0.054
JPN	-0.013	0.098	-	0.220	-0.053	0.251	0.069	0.095
GER	0.166	0.185	0.220	-	0.108	0.123	0.157	0.160
UK	-0.146	0.007	-0.053	0.108	-	0.042	0.224	0.030
FRA	-0.007	0.105	0.251	0.123	0.042	-	0.085	0.100
ITA	-0.037	0.153	0.069	0.157	0.224	0.085	-	0.108
Average								0.072

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Average								0.072

Brandt, Cochrane, and Santa-Clara puzzle

Volatility of real exchange rate growth

	US	CAN	JPN	UK	GER	FRA	ITA	Average
US	-	4.506	13.131	10.965	12.479	11.629	11.620	10.722
CAN	4.506	-	13.615	11.435	13.127	12.422	12.365	12.593
JPN	13.131	13.615	-	12.585	11.597	11.470	13.038	12.461
UK	10.965	11.435	12.585	-	9.784	8.967	9.540	10.462
GER	12.479	13.127	11.597	9.784	-	4.631	8.804	9.588
FRA	11.629	12.422	11.470	8.967	4.631	-	7.003	8.898
ITA	11.620	12.365	13.038	9.540	8.804	7.003	-	10.150
Average								10.696

Brandt, Cochrane, and Santa-Clara puzzle

Volatility of real exchange rate growth

	US	CAN	JPN	UK	GER	FRA	ITA	Average
US	-	4.506	13.131	10.965	12.479	11.629	11.620	10.722
CAN	4.506	-	13.615	11.435	13.127	12.422	12.365	12.593
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UK	10.965	11.435	12.585	-	9.784	8.967	9.540	10.462
GER	12.479	13.127	11.597	9.784	-	4.631	8.804	9.588
FRA	11.629	12.422	11.470	8.967	4.631	-	7.003	8.898
ITA	11.620	12.365	13.038	9.540	8.804	7.003	-	10.150
Average								10.696

Markets' Sharpe Ratios

	US	CAN	JPN	GER	UK	FRA	ITA	Average
Sharpe ratio	58.694	31.278	22.038	43.169	46.286	40.904	22.099	37.781

Correlations of fundamentals

$$corr(\Delta c, \Delta c^*)$$

	US	CAN	JPN	GER	UK	FRA	ITA	Average
US	-	0.356	0.077	0.242	0.214	0.266	-0.018	0.228
CAN	0.356	-	-0.074	0.094	0.259	0.165	0.099	0.150
JPN	0.077	-0.074	-	0.067	0.054	0.151	0.126	0.067
GER	0.242	0.094	0.067	-	0.115	-0.066	-0.068	0.064
UK	0.214	0.259	0.054	0.115	-	0.311	-0.001	0.158
FRA	0.266	0.165	0.151	-0.066	0.311	-	0.134	0.160
ITA	-0.018	0.099	0.126	-0.068	-0.001	0.134	-	0.045
Average								0.125

Correlations of fundamentals

$$corr(\Delta c, \Delta c^*)$$

	US	CAN	JPN	GER	UK	FRA	ITA	Average
US	-	0.356	0.077	0.242	0.214	0.266	-0.018	0.228
CAN	0.356	-	-0.074	0.094	0.259	0.165	0.099	0.150
JPN	0.077	-0.074	-	0.067	0.054	0.151	0.126	0.067
GER	0.242	0.094	0.067	-	0.115	-0.066	-0.068	0.064
UK	0.214	0.259	0.054	0.115	-	0.311	-0.001	0.158
FRA	0.266	0.165	0.151	-0.066	0.311	-	0.134	0.160
ITA	-0.018	0.099	0.126	-0.068	-0.001	0.134	-	0.045
Average								0.125

$$corr(\Delta d, \Delta d^*)$$

	US	CAN	JPN	GER	UK	FRA	ITA	Average
US	-	0.151	0.014	-0.020	-0.053	0.019	0.045	0.026
CAN	0.151	-	0.057	0.173	0.255	-0.035	-0.110	0.082
JPN	0.014	0.057	-	-0.002	0.076	0.062	0.239	0.074
GER	-0.020	0.173	-0.002	-	0.203	0.202	0.183	0.123
UK	-0.053	0.255	0.076	0.203	-	0.219	0.244	0.157
FRA	0.019	-0.035	0.062	0.202	0.219	-	0.343	0.135
ITA	0.045	-0.110	0.239	0.183	0.244	0.343	-	0.157
Average								0.108

Correlations of returns

Correlations of stock markets' returns

	US	CAN	JPN	GER	UK	FRA	ITA	Average
US	-	0.821	0.481	0.487	0.637	0.588	0.433	0.574
CAN	0.821	-	0.392	0.428	0.540	0.486	0.407	0.512
JPN	0.481	0.392	-	0.294	0.422	0.359	0.389	0.389
GER	0.487	0.428	0.294	-	0.478	0.630	0.498	0.469
UK	0.637	0.540	0.422	0.478	-	0.541	0.376	0.499
FRA	0.588	0.486	0.359	0.630	0.541	-	0.587	0.532
ITA	0.433	0.407	0.389	0.498	0.376	0.587	-	0.448
Average								0.489

Correlations of returns

Correlations of stock markets' returns

	US	CAN	JPN	GER	UK	FRA	ITA	Average
US	-	0.821	0.481	0.487	0.637	0.588	0.433	0.574
CAN	0.821	-	0.392	0.428	0.540	0.486	0.407	0.512
JPN	0.481	0.392	-	0.294	0.422	0.359	0.389	0.389
GER	0.487	0.428	0.294	-	0.478	0.630	0.498	0.469
UK	0.637	0.540	0.422	0.478	-	0.541	0.376	0.499
FRA	0.588	0.486	0.359	0.630	0.541	-	0.587	0.532
ITA	0.433	0.407	0.389	0.498	0.376	0.587	-	0.448
Average								0.489

Correlations of risk-free rate

	US	CAN	JPN	GER	UK	FRA	ITA	Average
US	-	0.695	0.664	0.502	0.709	0.520	0.625	0.619
CAN	0.695	-	0.758	0.686	0.754	0.719	0.671	0.714
JPN	0.664	0.758	-	0.609	0.659	0.648	0.784	0.687
GER	0.502	0.686	0.609	-	0.630	0.701	0.563	0.615
UK	0.709	0.754	0.659	0.630	-	0.761	0.737	0.708
FRA	0.520	0.719	0.648	0.701	0.761	-	0.837	0.698
ITA	0.625	0.671	0.784	0.563	0.737	0.837	-	0.703
Average								0.678

Setup of the economy

- Complete markets

Setup of the economy

- Complete markets
- Agents have risk-sensitive preferences:

$$
\begin{aligned}
U_t &= (1-\delta)\log C_t + \frac{\delta}{1-\gamma}\log E_t \exp\left\{(1-\gamma)U_{t+1}\right\} \\
U_t^* &= (1-\delta)\log C_t^* + \frac{\delta}{1-\gamma}\log E_t \exp\left\{(1-\gamma)U_{t+1}^*\right\}
\end{aligned}
$$

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U_t^* &= (1-\delta)\log C_t^* + \frac{\delta}{1-\gamma}\log E_t \exp\{(1-\gamma)U_{t+1}^*\}
\end{aligned}
$$

- Long-run risks in consumption

$$
\begin{aligned}
\Delta c_{t+1} &= \mu_c + \lambda_1 z_{1,t} + \lambda_2 z_{2,t} + \sigma_c \varepsilon_{t+1} \\
\Delta c_{t+1}^* &= \mu_c^* + \lambda_2 z_{1,t} + \lambda_1 z_{2,t} + \sigma_c^* \varepsilon_{t+1}^* \\
z_{1,t} &= \rho_1 z_{1,t-1} + \sigma_1 \varepsilon_{1,t+1} \\
z_{2,t} &= \rho_2 z_{2,t-1} + \sigma_2 \varepsilon_{2,t+1}
\end{aligned}
$$

Calibration

$$U_t = (1-\delta)\log C_t + \frac{\delta}{1-\gamma}\log E_t \exp\{(1-\gamma)U_{t+1}\}$$

$$\begin{aligned}
\Delta c_{t+1} &= \mu_c + \lambda_1 z_{1,t} + \lambda_2 z_{2,t} + \sigma_c \varepsilon_{t+1} \\
\Delta c^*_{t+1} &= \mu^*_c + \lambda_2 z_{1,t} + \lambda_1 z_{2,t} + \sigma^*_c \varepsilon^*_{t+1} \\
z_{1,t} &= \rho_1 z_{1,t-1} + \sigma_1 \varepsilon_{1,t+1} \\
z_{2,t} &= \rho_2 z_{2,t-1} + \sigma_2 \varepsilon_{2,t+1}
\end{aligned}$$

Calibration

$$U_t = (1 - .991)\log C_t + \frac{.991}{1-8}\log E_t \exp\{(1-8)U_{t+1}\}$$

$$\begin{aligned}
\Delta c_{t+1} &= \mu_c + \lambda_1 z_{1,t} + \lambda_2 z_{2,t} + \sigma_c \varepsilon_{t+1} \\
\Delta c^*_{t+1} &= \mu^*_c + \lambda_2 z_{1,t} + \lambda_1 z_{2,t} + \sigma^*_c \varepsilon^*_{t+1} \\
z_{1,t} &= \rho_1 z_{1,t-1} + \sigma_1 \varepsilon_{1,t+1} \\
z_{2,t} &= \rho_2 z_{2,t-1} + \sigma_2 \varepsilon_{2,t+1}
\end{aligned}$$

Calibration

$$U_t = (1-.991)\log C_t + \frac{.991}{1-8}\log E_t \exp\{(1-8)U_{t+1}\}$$

$$\Delta c_{t+1} = .001 + \lambda_1 z_{1,t} + \lambda_2 z_{2,t} + .006\varepsilon_{t+1}$$
$$\Delta c^*_{t+1} = .001 + \lambda_2 z_{1,t} + \lambda_1 z_{2,t} + .006\varepsilon^*_{t+1}$$
$$z_{1,t} = .987 \cdot z_{1,t-1} + .00005\varepsilon_{1,t+1}$$
$$z_{2,t} = .987 \cdot z_{2,t-1} + .00005\varepsilon_{2,t+1}$$

Calibration

$$U_t = (1-.991)\log C_t + \frac{.991}{1-8}\log E_t \exp\{(1-8)U_{t+1}\}$$

$$\begin{aligned}
\Delta c_{t+1} &= .001 + 5.5 \cdot z_{1,t} + 1 \cdot z_{2,t} + .006\varepsilon_{t+1} \\
\Delta c^*_{t+1} &= .001 + 1 \cdot z_{1,t} + 5.5 \cdot z_{2,t} + .006\varepsilon^*_{t+1} \\
z_{1,t} &= .987 \cdot z_{1,t-1} + .00005\varepsilon_{1,t+1} \\
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$$U_t = (1-.991)\log C_t + \frac{.991}{1-8}\log E_t \exp\{(1-8)U_{t+1}\}$$

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\Delta c_{t+1} &= .001 + 5.5 \cdot z_{1,t} + z_{2,t} + .006\varepsilon_{t+1} \\
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z_{2,t} &= .987 \cdot z_{2,t-1} + .00005\varepsilon_{2,t+1}
\end{aligned}$$

$$corr\left(\varepsilon_{t+1}, \varepsilon^*_{t+1}\right) = 0.1$$

Calibration

$$U_t = (1-.991)\log C_t + \frac{.991}{1-8}\log E_t \exp\{(1-8)U_{t+1}\}$$

$$\begin{aligned}
\Delta c_{t+1} &= .001 + 5.5 \cdot z_{1,t} + z_{2,t} + .006\varepsilon_{t+1} \\
\Delta c^*_{t+1} &= .001 + z_{1,t} + 5.5 \cdot z_{2,t} + .006\varepsilon^*_{t+1} \\
z_{1,t} &= .987 \cdot z_{1,t-1} + .00005\varepsilon_{1,t+1} \\
z_{2,t} &= .987 \cdot z_{2,t-1} + .00005\varepsilon_{2,t+1}
\end{aligned}$$

$$corr\left(\varepsilon_{t+1}, \varepsilon^*_{t+1}\right) = 0.1, \quad corr\left(\varepsilon_{1,t+1}, \varepsilon_{2,t+1}\right) = 0.6$$

Calibration

$$U_t = (1-.991)\log C_t + \frac{.991}{1-8}\log E_t \exp\{(1-8)U_{t+1}\}$$

$$\begin{aligned}
\Delta c_{t+1} &= .001 + 5.5 \cdot z_{1,t} + z_{2,t} + .006\varepsilon_{t+1} \\
\Delta c^*_{t+1} &= .001 + z_{1,t} + 5.5 \cdot z_{2,t} + .006\varepsilon^*_{t+1} \\
z_{1,t} &= .987 \cdot z_{1,t-1} + .00005\varepsilon_{1,t+1} \\
z_{2,t} &= .987 \cdot z_{2,t-1} + .00005\varepsilon_{2,t+1}
\end{aligned}$$

$$corr\left(\varepsilon_{t+1}, \varepsilon^*_{t+1}\right) = 0.1, \quad corr\left(\varepsilon_{1,t+1}, \varepsilon_{2,t+1}\right) = 0.6$$

$$corr\left(\varepsilon_{t+1}, \varepsilon_{1,t+1}\right) = 0.06, \quad corr\left(\varepsilon^*_{t+1}, \varepsilon_{2,t+1}\right) = 0.06$$

Calibration

$$U_t = (1-.991)\log C_t + \frac{.991}{1-8}\log E_t \exp\{(1-8)U_{t+1}\}$$

$$\begin{aligned}
\Delta c_{t+1} &= .001 + 5.5\cdot z_{1,t} + z_{2,t} + .006\varepsilon_{t+1} \\
\Delta c^*_{t+1} &= .001 + z_{1,t} + 5.5\cdot z_{2,t} + .006\varepsilon^*_{t+1} \\
z_{1,t} &= .987\cdot z_{1,t-1} + .00005\varepsilon_{1,t+1} \\
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\end{aligned}$$

$$corr\left(\varepsilon_{t+1}, \varepsilon^*_{t+1}\right) = 0.1, \quad corr\left(\varepsilon_{1,t+1}, \varepsilon_{2,t+1}\right) = 0.6$$
$$corr\left(\varepsilon_{t+1}, \varepsilon_{1,t+1}\right) = 0.06, \quad corr\left(\varepsilon^*_{t+1}, \varepsilon_{2,t+1}\right) = 0.06$$
$$corr\left(\varepsilon_{t+1}, \varepsilon_{2,t+1}\right) = 0.60, \quad corr\left(\varepsilon^*_{t+1}, \varepsilon_{1,t+1}\right) = 0.60$$

Explaining the Backus and Smith anomaly

Explaining the Backus and Smith anomaly

$$\Delta e_t - E_{t-1}\left[\Delta e_t\right] = \left(\varepsilon_{c,t} - \varepsilon_{c^*,t}\right)$$

$$\left(\Delta c_t - \Delta c_t^*\right) - E_{t-1}\left(\Delta c_t - \Delta c_t^*\right) = \left(\varepsilon_{c,t} - \varepsilon_{c^*,t}\right)$$

Explaining the Backus and Smith anomaly

$$\Delta e_t - E_{t-1}[\Delta e_t] = (\varepsilon_{c,t} - \varepsilon_{c^*,t}) + \left[(1-\gamma)\frac{\delta(\lambda_2 - \lambda_1)}{1-\delta\rho}\right](\varepsilon_{1,t} - \varepsilon_{2,t})$$

$$(\Delta c_t - \Delta c_t^*) - E_{t-1}(\Delta c_t - \Delta c_t^*) = (\varepsilon_{c,t} - \varepsilon_{c^*,t})$$

Explaining the Backus and Smith anomaly

$$\Delta e_t - E_{t-1}\left[\Delta e_t\right] = \left(\varepsilon_{c,t} - \varepsilon_{c^*,t}\right) + \underbrace{\left[(1-8)\frac{\delta(1-5)}{1-\delta\rho}\right]}_{>0}\left(\varepsilon_{1,t} - \varepsilon_{2,t}\right)$$

$$\left(\Delta c_t - \Delta c_t^*\right) - E_{t-1}\left(\Delta c_t - \Delta c_t^*\right) = \left(\varepsilon_{c,t} - \varepsilon_{c^*,t}\right)$$

Explaining the Backus and Smith anomaly

$$\Delta e_t - E_{t-1}[\Delta e_t] = (\varepsilon_{c,t} - \varepsilon_{c^*,t}) + \underbrace{\left[(1-8)\frac{\delta(1-5)}{1-\delta\rho}\right]}_{>0}(\varepsilon_{1,t} - \varepsilon_{2,t})$$

$$(\Delta c_t - \Delta c_t^*) - E_{t-1}(\Delta c_t - \Delta c_t^*) = (\varepsilon_{c,t} - \varepsilon_{c^*,t})$$

Can we engineer negative correlation between $(\varepsilon_{c,t} - \varepsilon_{c^*,t})$ and $(\varepsilon_{1,t} - \varepsilon_{2,t})$?

Explaining the Backus and Smith anomaly

$$\Delta e_t - E_{t-1}[\Delta e_t] = (\varepsilon_{c,t} - \varepsilon_{c^*,t}) + \underbrace{\left[(1-8)\frac{\delta(1-5)}{1-\delta\rho}\right]}_{>0}(\varepsilon_{1,t} - \varepsilon_{2,t})$$

$$(\Delta c_t - \Delta c_t^*) - E_{t-1}(\Delta c_t - \Delta c_t^*) = (\varepsilon_{c,t} - \varepsilon_{c^*,t})$$

Can we engineer negative correlation between $(\varepsilon_{c,t} - \varepsilon_{c^*,t})$ and $(\varepsilon_{1,t} - \varepsilon_{2,t})$?

Yes, by setting:

- high correlation b/w $\varepsilon_{c,t}$ and $\varepsilon_{2,t}$ and b/w $\varepsilon_{c,t}^*$ and $\varepsilon_{1,t}$

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Yes, by setting:

- high correlation b/w $\varepsilon_{c,t}$ and $\varepsilon_{2,t}$ and b/w $\varepsilon_{c,t}^*$ and $\varepsilon_{1,t}$
- low correlation b/w $\varepsilon_{c,t}$ and $\varepsilon_{1,t}$ and b/w $\varepsilon_{c,t}^*$ and $\varepsilon_{2,t}$

Explaining the Brandt, Cochrane, and Santa-Clara anomaly (and its side-effects!)

Low real exchange rate volatility

$$\Delta e_t - E_{t-1}[\Delta e_t] = \underbrace{(\varepsilon_{c,t} - \varepsilon_{c^*,t})}_{\substack{\textbf{Data}:\ \varepsilon_{c,t}\text{ and }\varepsilon^*_{c,t}\\ \text{have low correlation}}} + \kappa \cdot \underbrace{(\varepsilon_{1,t} - \varepsilon_{2,t})}_{\substack{\textbf{Colacito \& Croce (2007)}:\\ \varepsilon_{1,t}\text{ and }\varepsilon_{2,t}\text{ must}\\ \text{have } \textbf{high} \text{ correlation}}}$$

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Drawback: are risk-free rates too correlated?

$$\begin{aligned} r^f_t - \bar{r}^f &= \lambda_1 z_{1,t} + \lambda_2 z_{2,t} \\ r^{f^*}_t - \bar{r}^{f^*} &= \lambda_2 z_{1,t} + \lambda_1 z_{2,t} \end{aligned}$$

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Can we fix this?

- Reduce $corr\left(\varepsilon_{1,t}, \varepsilon_{2,t}\right)$

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Can we fix this?

- Reduce $corr(\varepsilon_{1,t}, \varepsilon_{2,t})$
- Turn on $corr(\varepsilon_{c,t}, \varepsilon_{2,t})$=$corr(\varepsilon_{c,t}^*, \varepsilon_{1,t})$=0.6

Remainder of the model

- Stochastic volatility:

$$\begin{aligned}
\Delta c_{t+1} &= \mu_c + \lambda_1 z_{1,t} + \lambda_2 z_{2,t} + \lambda_t^{1/2} \varepsilon_{t+1} \\
\Delta c^*_{t+1} &= \mu_c + \lambda_1^* z_{1,t} + \lambda_2^* z_{2,t} + \lambda_t^{*1/2} \varepsilon^*_{t+1} \\
z_{1,t} &= \rho_1 z_{1,t-1} + \lambda_{t-1}^{1/2} \varepsilon_{1,t+1} \\
z_{2,t} &= \rho_2 z_{2,t-1} + \lambda_{t-1}^{*1/2} \varepsilon_{2,t+1} \\
\lambda_t &= \sigma (1 - \rho_\lambda) + \rho_\lambda \lambda_{t-1} + \varphi_\lambda \lambda_{t-1} \varepsilon_{\lambda,t} \\
\lambda_t^* &= \sigma (1 - \rho_\lambda) + \rho_\lambda \lambda^*_{t-1} + \varphi_\lambda \lambda^*_{t-1} \varepsilon^*_{\lambda,t}
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Remainder of the model

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\lambda_t^* &= \sigma(1-\rho_\lambda) + \rho_\lambda \lambda^*_{t-1} + \varphi_\lambda \lambda^*_{t-1} \varepsilon^*_{\lambda,t}
\end{aligned}
$$

- Cash flows of redundant assets:

$$
\begin{aligned}
\Delta d_{t+1} &= \mu_d + \lambda_{d_1} z_{1,t} + \lambda_{d_2} z_{2,t} + \lambda_t^{1/2} \varepsilon_{d,t+1} \\
\Delta d^*_{t+1} &= \mu_d + \lambda^*_{d_1} z_{1,t} + \lambda^*_{d_2} z_{2,t} + \lambda_t^{*1/2} \varepsilon^*_{d,t+1}
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Further results

- Well known results:
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High Correlation
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Low Correlation
$$\begin{cases} \Delta d_t - E_{t-1}\left[\Delta d_t\right] = \varepsilon_{d,t} \\ \Delta d^*_t - E_{t-1}\left[\Delta d^*_t\right] = \varepsilon^*_{d,t} \end{cases}$$

Results

	Data	Model	95% CI
Volatility of exchange rate growth	10.696	10.929	(9.105,12.954)
UIP regression slope	-0.136	-0.062	(-0.609,0.458)
Sharpe ratio	37.781	40.564	(2.783,76.447)
Backus and Smith anomaly	0.072	-0.056	(-0.279,0.165)
Correlation of risk-free rates	0.678	0.793	(0.189,0.954)
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AC(2)	0.191	0.158	(-0.154,0.484)
AC(3)	0.158	0.140	(-0.161,0.473)
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Correlation of consumption growths	0.125	0.247	(-0.032,0.546)

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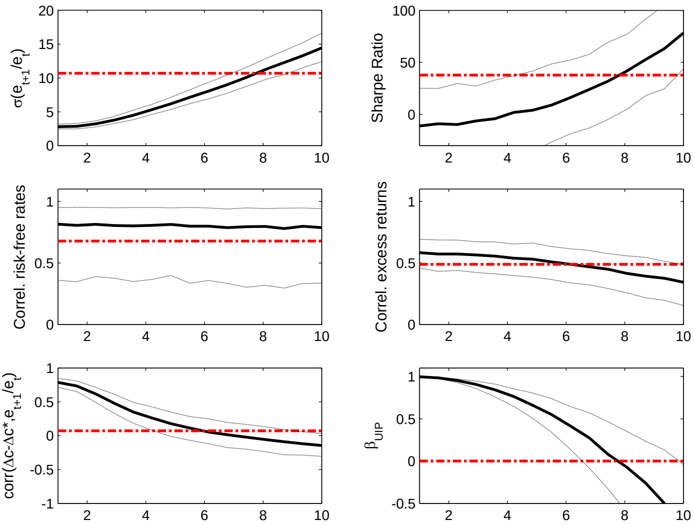
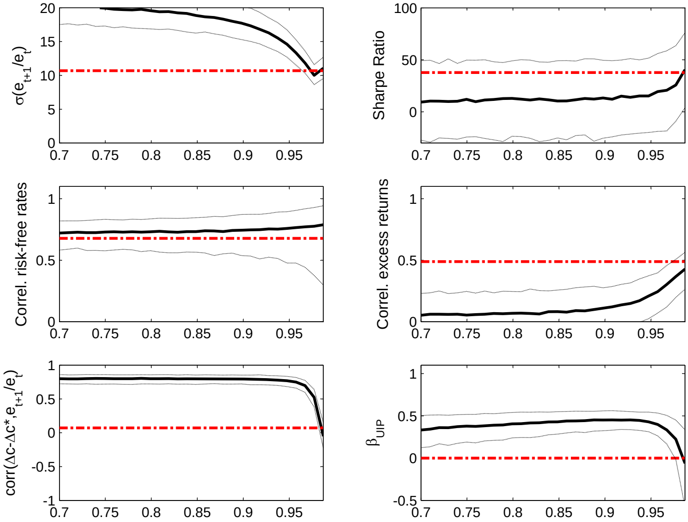
Volatility of dividend growth	9.329	9.845	(8.115,12.045)
AC(1)	0.158	0.109	(-0.142,0.384)
AC(2)	0.093	0.088	(-0.161,0.376)
AC(3)	0.086	0.087	(-0.161,0.358)
AC(4)	0.182	0.075	(-0.164,0.356)
Correlation of dividend growths	0.108	0.135	(-0.113,0.380)

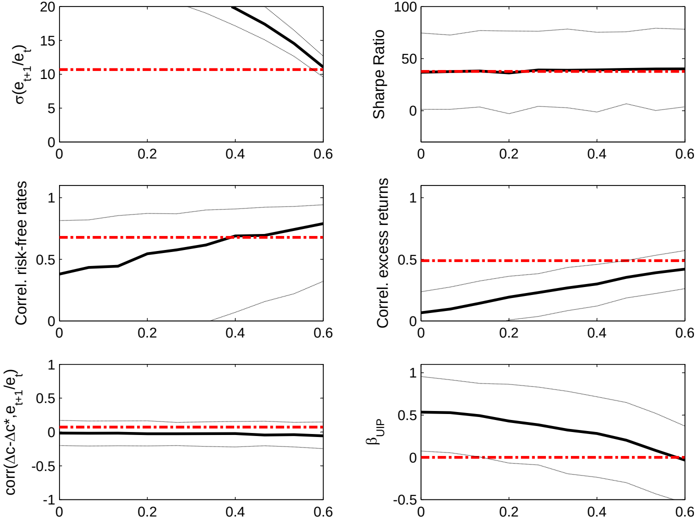
The case for risk-sensitive preferences



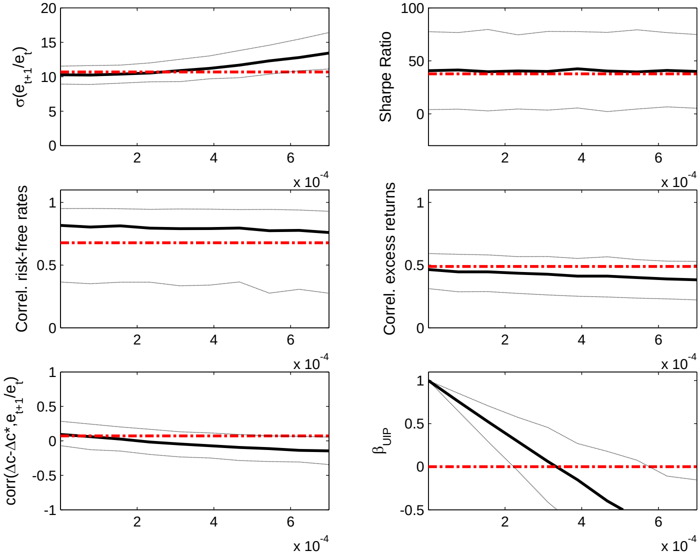
The case for long-run risks



The case for two factors



The case for stochastic volatility



Concluding remarks

- A simple two-country long-run risks model can explain
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  2. Brandt, Cochrane, and Santa-Clara puzzle
  3. Negative slope in UIP regressions
  4. High correlation of stock markets
  5. High (but not perfect!) correlation of risk-free rates
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  - Identification of low frequency components of consumption growth: Colacito, Croce, and Ghysels (2008)