Risk Sharing in International Economies and Market Incompleteness

by Bakshi, Cerrato, and Cosby

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Roadmap

1. Reinterpreting the Results
2. Relation to the Literature
3. My comments

SDF correlation is not the same as Risk Sharing

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- Two countries, two goods, log- time-additive preferences

$$
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U_{f,t} &= (1-\delta)\log(x_{f,t}^{1-\alpha} \cdot y_{f,t}^{\alpha}) + \delta E_t[U_{f,t+1}]
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where $\alpha > 1/2$ captures home bias.

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- Markets are complete: perfect risk sharing can be achieved

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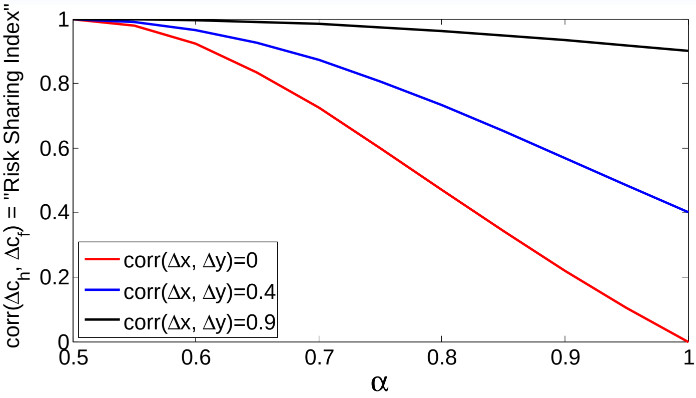
- What do the correlation of SDF's and the "Risk-Sharing Index" look like?

Risk-Sharing Index?



- For simplicity, assume $\sigma[\Delta x] = \sigma[\Delta y] = 0.02$.
- It follows that $RSI = corr(\Delta c_h, \Delta c_f)$.

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Restrictions on correlation of SDF, not Risk Sharing

- We cannot say anything about risk sharing from looking at SDF correlations, unless we make specific assumptions about the economic model.
- This paper provides restrictions on the correlation of SDFs in the presence of one form of market incompleteness.
- This is still valuable to refine the set of economic models that can explain the dynamics of intenrational asset prices and quantities.

Relation to the literature

In complete markets, by no arbitrage

$$\Delta e_{t+1} = m^*_{t+1} - m_{t+1}$$

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$$V[\Delta e] = V[m^*] + V[m] - 2\rho_{m^*,m}\sqrt{V[m^*]\,V[m]}$$

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1. Asset Prices:

$$\underbrace{V[\Delta e]}_{1.5\%} = \underbrace{V[m^*]}_{20\%} + \underbrace{V[m]}_{20\%} - 2\underbrace{\rho_{m^*,m}}_{0.96}\underbrace{\sqrt{V[m^*]\,V[m]}}_{20\%}$$

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$$m_{t+1} = -\gamma\Delta c_{t+1} \qquad \text{[CRRA preferences]}$$

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Relation to the literature (cont'd)

- Economic models that deliver high correlation of SDFs from consumption data: Colacito and Croce (JPE, 2011), Colacito and Croce (JF, 2013), Verdelhan (JF, 2010), Stathopulos (2014), Fahri and Gabaix (WP, 2015), ...
- This paper: lower the correlation of SDF through market incompleteness.

This paper

$$(P_t/e_t - P_t^*) \quad = \quad E\left[\left(M_{t+1}\frac{e_{t+1}}{e_t} - M_{t+1}^*\right)\left(P_{t+1}^* + D_{t+1}^*\right)\right]$$

- Difference of two Euler equations

This paper

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$$\left(P_t/e_t - P_t^*\right)/P_t^* \quad = \quad E\left[\left(M_{t+1}\frac{e_{t+1}}{e_t} - M_{t+1}^*\right)\left(P_{t+1}^* + D_{t+1}^*\right)/P_t^*\right]$$

- Divide both sides by P_t^*

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- Divide both sides by P_t^*
- RHS: foreign market return
- LHS: return from buying the asset in the foreign market and selling it in the home market

This paper

$$\left(P_t/e_t - P_t^*\right)/P_t^* \quad = \quad cov\left[\left(M_{t+1}\frac{e_{t+1}}{e_t} - M_{t+1}^*\right), R_{t+1}^*\right]$$

- Assume that $E\left[M_{t+1}\frac{e_{t+1}}{e_t} - M_{t+1}^*\right] = 0$

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$$\left(P_t/e_t - P_t^*\right)/P_t^* \quad \leq \quad \sigma\left(M_{t+1}\frac{e_{t+1}}{e_t} - M_{t+1}^*\right)\cdot\sigma\left(R_{t+1}^*\right)$$

- Assume that $E\left[M_{t+1}\frac{e_{t+1}}{e_t} - M_{t+1}^*\right] = 0$
- Use the Cauchy-Schwartz inequality

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- In complete markets: $M_{t+1}\frac{e_{t+1}}{e_t} - M_{t+1}^* = 0$.
- The prices of a claim to D^* are equalized after adjusting for currency: $P_t/e_t = P_t^*$.

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$$\frac{\left(P_t/e_t - P_t^*\right)/P_t^*}{\sigma\left(R_{t+1}^*\right)} \leq \sigma\left(M_{t+1}\frac{e_{t+1}}{e_t} - M_{t+1}^*\right) \leq \Theta$$

- In incomplete markets, they “bound good deals”

This paper

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- In incomplete markets, they “bound good deals”
- Example:
  - $P^* = GBP\ 4$, $P/e = GBP\ 5$, $Var(R^*) = 15\%$
  - reward to risk ratio is 166%
  - A great deal!

This paper

$$\frac{\left(P_t/e_t - P_t^*\right)/P_t^*}{\sigma\left(R_{t+1}^*\right)} \leq \sigma\left(M_{t+1}\frac{e_{t+1}}{e_t} - M_{t+1}^*\right) \leq \Theta$$

- What should we expect?

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- What should we expect?
- If $\Theta \uparrow$, the correlation between M and M^* should drop.

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- What should we expect?
- If $\Theta \uparrow$, the correlation between M and M^* should drop.
- Economic intuition: if the price of the same asset in two financial markets is different, investors' discount rates are less than perfectly aligned.

Results

- Page 14: Germany, France, Netherlands, and Switzerland have high values of the *correlation of SDFs*
- Page 15: for country pairs with high interest differentials, the *correlation of SDFs* is low

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	AUS	NZ	UK	FRA	CAN	US	NED	GER	JPN	SWI
AUS	-	54.2	67.7	62.3	61.2	70.5	74.4	63.4	55.7	74.3
NZ		-	65.9	54	54.6	68	73.1	53.2	**28.9**	73
UK			-	69.4	69.8	74.2	76.9	71.6	67.5	77.3
FRA				-	62.1	70.8	78.9	71.4	55.5	77.4
CAN					-	71.5	75.5	61.6	55.2	74.9
US						-	76.7	70.7	70.9	76.8
NED							-	78.6	75	79.7
GER								-	56.5	77.4
JPN									-	75.4
SWI										-

RSI/correlations for Θ=0.35

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- Page 15: for country pairs with high interest differentials, the *correlation of SDFs* is low
- Is this necessarily a reflection of incomplete markets/poor risk sharing across countries?
- What if we assume complete markets and heterogeneous exposure to global risk?

  $\hookrightarrow$ Lustig, Roussanov, and Verdelhan (RFS, 2011); Hassan and Mano (2015); Colacito, Croce, Gavazzoni, and Ready (2015); ...

Reinterpreting the Results

$$
\begin{aligned}
m &= \overline{m} - (\lambda + \Delta_\lambda) \cdot \varepsilon_{global} - \varepsilon \\
m^* &= \overline{m}^* - (\lambda - \Delta_\lambda) \cdot \varepsilon_{global} - \varepsilon^*
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- High exposure $(\lambda + \Delta_\lambda)$: Japan

  ↪ low interest rate

  ↪ currency appreciates in bad times

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- High exposure ($\lambda + \Delta_\lambda$): Japan

  $\hookrightarrow$ low interest rate

  $\hookrightarrow$ currency appreciates in bad times

- Low exposure ($\lambda - \Delta_\lambda$): Australia/New Zealand

  $\hookrightarrow$ high interest rate

  $\hookrightarrow$ currency depreciates in bad times

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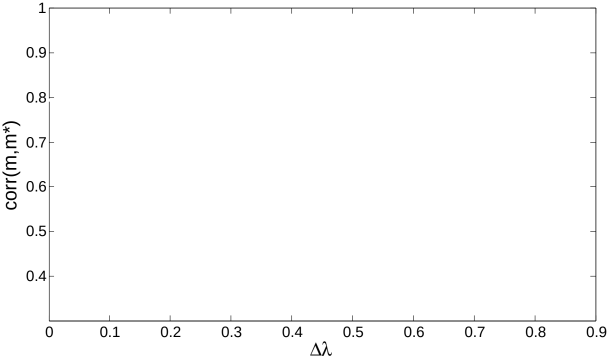

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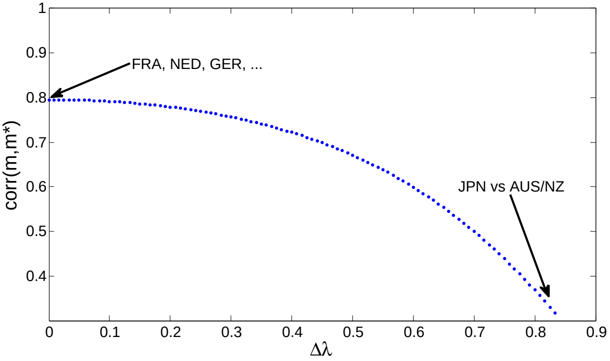

Concluding remarks

- An interesting paper
- A window into how correlated int'l SDFs should be in the presence of market incompleteness