Six anomalies looking for a model.

A consumption based explanation of international finance puzzles

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Abstract

When the dynamics of consumption growth in two countries include small and highly persistent predictable components and agents display a preference for the timing of the resolution of uncertainty, it is possible to explain a large number of international finance puzzles. This includes the relatively small volatility of the growth rate of the US dollar, the high correlation of stock markets returns paired with a modest degree of co-movement of fundamentals and the low correlation between consumption growth differentials and the depreciation of the exchange rate. When stochastic volatility is also assumed to be a factor in accounting for the dynamics of consumption growth, then the forward premium anomaly can also be accounted for by the model.

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1 Introduction

One of the most well document facts of international finance is the failure of the traditional consumption based asset pricing model to explain many of the features of the joint distribution of prices and quantities. Backus and Smith (1993) pointed out that consumption differentials across countries and exchange rates are almost uncorrelated in a large cross-section of countries, a finding at odds with most models. The considerable equity premium and the low volatility of consumption growth that we observe in major industrialized countries extends the puzzle that Mehra and Prescott (1985) pointed out for the United States to a larger set of countries. The low correlation of consumption between the same set of countries opens up to an international equity premium puzzle, that comes in the form of a relatively smooth exchange rate, as suggested by Brandt, Cochrane and Santa-Clara (2006). Bansal and Lundblad (2002) observe that the almost complete lack of international correlation between fundamentals poses serious difficulties for any asset pricing model to explain the moderate to high correlation between international stock markets. Fama (1984) documents the tendency for high interest rate currencies to appreciate, when the traditional consumption based asset pricing model might lead us to think the opposite. In this paper we provide an equilibrium model that can simultaneously account for six of these anomalies.

We assume that agents have a preference for the timing of the resolution of uncertainty in the sense of Epstein and Zin (1989). This means that shocks that have a long-lasting impact on consumption growth are likely to produce large movements in stochastic discount factors. Indeed, Bansal and Yaron (2004) have shown that allowing the dynamics of consumption growth to be driven not only by temporary (short-run) shocks, but also by persistent (long-run) shocks is successful in accounting for the large equity premium that we observed in the US over the past century.[1] Colacito and Croce (2006) have documented that a two country version of this setup is able to explain some important stylized facts of the joint distribution of international stock market returns and exchange rates. They obtain these results in the context of a

[1]Specifically, persistent shocks come in the form of shocks to a predictive component of consumption growth and as such only affect the level of consumption with a one period delay. We retain this assumption in this paper.

complete markets economy.

In this paper we extend the economy set forward by Colacito and Croce (2006) in two important directions. First, we let two persistent sources of risk enter the dynamics of consumption growth in each country. We assume that the long-run components affect consumption growth in the two countries asymmetrically, as good news to one factor will lead to a larger increase in consumption in one country relative to the other. Second, we show that the specific structure of the correlation of consumption shocks both within and across countries is relevant in explaining a large set of international finance anomalies.

We document that this setup is consistent with the joint dynamics of consumption that we observe in G-7 countries over the last 30 years. In economic terms, a larger exposure of consumption to a specific source of long-run risk can be interpreted as the result of a preference bias toward a specific good. Colacito and Croce (2008) provide general equilibrium foundations supporting this prediction.

Backus, Foresi and Telmer (2001) have shown that when markets are complete, the growth of the exchange rate between the currencies of two countries is equal to the difference of the stochastic discount factors between the same set of countries. Hence, the higher the degree of co-movement of intertemporal marginal rates of substitutions across countries, the smoother exchange rates will be.

We show that the combinations of two correlations is capable of reproducing the degree of volatility of exchange rate movements between major currencies. First, we assume that long-run risks are positively correlated. This channel was already featured in Colacito and Croce (2006), but would still result in too volatile exchange rates in a setup with multiple predictive components of consumption growth. This leads to the need for a second channel that comes in the form of a positive correlation between the short-run shock of one country's consumption growth and the long-run shock to the predictive component that has the largest predictive power for consumption growth in the other country. That is, if good (bad) news to consumption growth in one country are likely to come at a time in which the other country is also experiencing good (bad) news, exchange rates need to fluctuate less in equilibrium. We document that this calibration is consistent with the low degree of international correlation of consumption growth that we observe in the data.

Furthermore, since a short-run shock to consumption growth in one country increases the likelihood of a long-run shock in the other country, exchange rates may appreciate or depreciate at a time in which the difference of consumption growth across countries increases. This means that the Backus and Smith (1993) anomaly is resolved in this model.

The advantage of introducing multiple predictive factors entering the dynamics of consumption growth with different loadings is that real risk-free rates are now less than perfectly correlated. Similarly, allowing for the predictive components of consumption growth to also enter the dynamics of dividend growth, enables the model to replicate the high degree of correlation of international stock markets, despite fundamentals being largely disconnected.

When stochastic volatility is introduced in this framework, it is shown that the model can also account for the less than unity coefficient that is typically obtained in the uncovered interest rate parity regressions. This result follows directly from the conditions set forward by Fama (1984). It is important to stress, however, that although necessary, stochastic volatility is per se not enough to replicate the magnitude of the coefficient that we obtain from the data. We show in the paper that the kind of preferences that we employ and that the persistence of the predictive factors play crucial roles as well. In a related paper, Bansal and Shaliastovich (2007) introduce stochastic volatility in the model of Colacito and Croce (2006) to study the dynamics of nominal term structures of the interest rates in major industrialized countries.

1.1 Organization

This paper is organized as follows. In the next section we document the main anomalies that we want to explain in the cross-section of G-7 countries. In the third section we introduce our model that relies on non-time separable preferences and low frequency components of consumption and dividends growths. The following two sections show the results from a calibrated economy and perform a sensitivity analysis that shows how important each of our assumptions is. The last section concludes the paper.

2 The model

2.1 Setup of the economy

There are two countries, *home* and *foreign* and we index with a superscript star foreign variables. The following four assumptions will be retained throughout this paper.

Assumption 1 (Markets)**.** *There is a complete set of assets both domestically and internationally.*

The assumption of complete markets is an important one. As proved by Backus et al. (2001) home and foreign marginal rates of substitution uniquely identify the process for the exchange rate growth, given this market structure.

Assumption 2 (Preferences)**.** *The two countries are each populated by a representative consumer with risk-sensitive preferences defined over the consumption aggregates* C_t *and* C_t^**:*

$$\begin{aligned} U_t &= (1-\delta)\log C_t + \delta\theta \log E_t \exp\left\{\frac{U_{t+1}}{\theta}\right\} \\ U_t^* &= (1-\delta)\log C_t^* + \delta\theta \log E_t \exp\left\{\frac{U_{t+1}^*}{\theta}\right\} \end{aligned} \tag{1}$$

with $\theta = \frac{1}{1-\gamma}$.

According to assumption 2 agents have risk-sensitive preferences defined over domestic and foreign consumption bundles. This specification is due to Hansen and Sargent (1995) and is used among others by Tallarini (2000) and Anderson (2005). The main departure from the constant relative risk-aversion case that is often analyzed in the literature lies in the fact that these preferences are non-time additive and they allow agents to be risk-averse in future utility in addition to future consumption.[2] Alterna-

[2]More precisely the CRRA case is nested in this specification as the limiting case in which $\gamma \to 1$.

tively they can be interpreted as the special case of Epstein and Zin (1989) preferences in which the intertemporal elasticity of substitution equals 1.

In this paper, we follow the approach of Mehra and Prescott (1985) in that we specify the consumption processes and derive the equilibrium dynamics of asset returns and exchange rates by looking at the Euler equation restrictions. Specifically we assume the following concerning the dynamics of consumption growth in the two countries.

Assumption 3 (Consumption dynamics)**.** *The logarithm of consumption follows a unit root process in each of the two countries:*

$$\begin{aligned} \log C_{t+1} - \log C_t &= \Delta c_{t+1} = \mu_c + \lambda_1 z_{1,t} + \lambda_2 z_{2,t} + \lambda_t^{1/2} \varepsilon_{t+1} \\ \log C^*_{t+1} - \log C^*_t &= \Delta c^*_{t+1} = \mu_c + \lambda^*_1 z_{1,t} + \lambda^*_2 z_{2,t} + {\lambda^*_t}^{1/2} \varepsilon^*_{t+1}, \quad \forall t \geq 0 \end{aligned} \tag{2}$$

where $z_{1,t}$ and $z_{2,t}$ are $AR(1)$ processes

$$\begin{aligned} z_{1,t} &= \rho_1 z_{1,t-1} + \lambda_{t-1}^{1/2} \varepsilon_{1,t+1} \\ z_{2,t} &= \rho_2 z_{2,t-1} + {\lambda^*_{t-1}}^{1/2} \varepsilon_{2,t+1} \end{aligned}$$

and λ_t and λ^*_t are stochastic variance processes with $AR(1)$ dynamics:

$$\begin{aligned} \lambda_t &= \sigma (1 - \rho_\lambda) + \rho_\lambda \lambda_{t-1} + \varphi_\lambda \lambda_{t-1} \varepsilon_{\lambda,t} \\ \lambda^*_t &= \sigma (1 - \rho_\lambda) + \rho_\lambda \lambda^*_{t-1} + \varphi_\lambda \lambda^*_{t-1} \varepsilon^*_{\lambda,t} \end{aligned} \tag{3}$$

Assumption 3 can be interpreted in several ways. First, this can be the outcome of an endowment economy in which there are two country-specific goods and preferences are defined only over the domestic one. In equilibrium, the dynamics of consumption equal those of the endowment process and prices can be read off the first order conditions.[3]. Alternatively, the process described in (2) can be thought of as the result of post-trade allocations. Colacito and Croce (2008) examine the dynamics of risk-sensitive allocations for the case in which the endowments of the two countries are specified as in (2) and agents have preferences defined over both goods.

The assumptions regarding $z_{1,t}$ and $z_{2,t}$ deserve additional comments. According to (2),

[3] For a proof of the existence of the exchange rate in this no-trade equilibrium refer to Colacito (2006).

a weighted sum of these two processes provides the conditional expectation of consumption growth in the two counties. This specification is a two-factor generalization of the process hypothesized by Colacito and Croce (2006) and extends the closed economy dynamics of Bansal and Yaron (2004). The debate as to whether consumption growth is i.i.d. or whether it can be predicted has been discussed in many papers, including Bansal, Kiku and Yaron (2006) and Colacito and Croce (2006). In this paper we document that the calibrated consumption dynamics are consistent with a large set of the observed moments of the joint distribution of consumption and dividend growth in G-7 countries.

A final remark on assumption (3) should be made about λ_t and λ_t^*. These country specific processes introduce time-varying uncertainty in the dynamics of consumption growths and of their predictable components. These stochastic volatility processes are assumed to be linear mainly for their tractability. We will show in a later section that all of the international facts can be explained in the context of a model that does not feature stochastic volatility, with the only exception of the empirical coefficient of the uncovered interest rate parity regressions. This is a well known theoretical result that dates back to Fama (1984). In some sense the long-run risk model is a sideshow of stochastic volatility when it comes to accounting for this coefficient, but the impact of stochastic volatility must be taken into account in the explanation of all other international variables.[4]

Assumption 4 (Dividend dynamics)**.** *Dividends in the two countries follow log-unit root processes:*

$$\begin{aligned} \Delta d_{t+1} &= \mu_d + \lambda_{d_1} z_{1,t} + \lambda_{d_2} z_{2,t} + \lambda_t^{1/2} \varepsilon_{d,t+1} \\ \Delta d_{t+1}^* &= \mu_d + \lambda_{d_1}^* z_{1,t} + \lambda_{d_2}^* z_{2,t} + \lambda_t^{*^{1/2}} \varepsilon_{d,t+1}^* \end{aligned} \tag{4}$$

Assumption 4 is a natural extension of the consumption growth dynamics. It has a tradition in the international finance literature starting with Bansal and Lundblad (2002) and more recently with Colacito and Croce (2006). These papers show that the presence of a time-varying trend in the dynamics of international dividends cannot be rejected. In discussing the implications of our model we will check that the moments

[4]For further discussion on long-run risks model and stochastic volatility refer to Bansal and Shaliastovich (2007).

of the distribution of dividends of the model are in line with the data on a large cross-section of countries.

2.2 Solution of the model

The relevant vector of state variables for this problem is $s_t = [z_{1,t}, z_{2,t}, \lambda_t, \lambda_t^*, h_t]$, where $h_t = \lambda_t^{1/2} \lambda_t^{*^{1/2}}$. In the appendix we show that the utility consumption ratio in each country can be expressed as a linear function of s_t:

$$\begin{aligned} V_t &= U_t - \log C_t = \Gamma_0 + \Gamma s_t' \\ V_t^* &= U_t^* - \log C_t^* = \Gamma_0^* + \Gamma^* s_t' \end{aligned}$$

where the entries of the vectors Γ_0, Γ_0^*, Γ and Γ^* are reported in the appendix.

The intertemporal marginal rates of substitutions impelled by the class of preferences used in this model are:[5]

$$\begin{aligned} M_{t+1} &= \frac{\partial U_t / \partial C_{t+1}}{\partial U_t / \partial C_t} = \delta \left(\frac{C_{t+1}}{C_t} \right)^{-1} \frac{\exp\left\{ \frac{U_{t+1}}{\theta} \right\}}{E_t \exp\left\{ \frac{U_{t+1}}{\theta} \right\}} \\ M_{t+1}^* &= \frac{\partial U_t^* / \partial C_{t+1}^*}{\partial U_t^* / \partial C_t^*} = \delta \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-1} \frac{\exp\left\{ \frac{U_{t+1}^*}{\theta} \right\}}{E_t \exp\left\{ \frac{U_{t+1}^*}{\theta} \right\}}, \quad \forall t \geq 0 \end{aligned} \tag{5}$$

Given the linear solution of the utility-log consumption ratios presented above, the logarithms of the stochastic discount factors are also linear functions of the state vector:

$$\begin{aligned} m_{t+1} &= \log M_{t+1} = -v_{0,m} - v_m s_t' + \frac{1}{\theta} \left(\widetilde{v}_1 \lambda_t^{1/2} + v_2 \lambda_t^{*^{1/2}} \right) \eta_{t+1}' \\ m_{t+1}^* &= \log M_{t+1}^* = -v_{0,m} - v_m^* s_t' + \frac{1}{\theta} \left(v_1^* \lambda_t^{1/2} + \widetilde{v}_2^* \lambda_t^{*^{1/2}} \right) \eta_{t+1}' \end{aligned} \tag{6}$$

with $\eta_t = \begin{bmatrix} \varepsilon_{1,t} & \varepsilon_{2,t} & \varepsilon_{c,t} & \varepsilon_{c^*,t} & \varepsilon_{\lambda,t} & \varepsilon_{\lambda^*,t} \end{bmatrix}$ *and* $v_{0,m}$, v_m, v_m^*, $\widetilde{v}_1$, v_1^*, v_2, $\widetilde{v}_2^*$ defined in the Appendix. Hence the model features time varying market prices of risk.

[5]We will use the words intertemporal marginal rate of substitution and stochastic discount factor interchangeably.

The rest of the solution of the model is obtained from no-arbitrage conditions and the assumption of complete markets. Following Backus et al. (2001), the logarithm of real exchange rate growth equals the difference of the log-stochastic discount factors:

$$\frac{e_{t+1}}{e_t} = m^*_{t+1} - m_{t+1}$$

Asset returns are obtained from the Euler equation restrictions:

$$\begin{aligned} E_t\left[\exp\{m_{t+1}\}R_{t+1}\right] &= 1 \\ E_t\left[\exp\{m^*_{t+1}\}R^*_{t+1}\right] &= 1 \end{aligned}$$

where $\log R_{t+1} = [r_{d,t+1}, r_{t+1}]'$ and $\log R^*_{t+1} = \left[r^*_{d,t+1}, r^*_{t+1}\right]'$ are the vectors of returns on the dividend paying asset and the risk-free asset, in the home and in the foreign country, respectively.

Given the log-linear dynamics of the stochastic discount factors, of the consumption and dividend growth processes, and of the vector of state variables s_t, the Appendix documents that the equilibrium dynamics of the system is accounted for by the following linear specification:

$$y_{t+1} = \Lambda_0 + \Lambda_1 s'_t + \left(H_y \lambda_t^{1/2} + H^*_y \lambda_t^{*^{1/2}}\right)\eta_{t+1}$$

where $y_{t+1} = \left[\Delta e_{t+1}, r_{t+1}, r^*_{t+1}, r_{d,t+1}, r^*_{d,t+1}, \Delta c_{t+1}, \Delta c^*_{t+1}, \Delta d_{t+1}, \Delta d^*_{t+1}\right]$. The Appendix reports all the details of the vectors and matrices of the above system. We will highlight the main features of the system in the next section.

2.3 Calibration

Table 6 reports the baseline calibration. The coefficient of risk aversion γ is set to 8, which is on the down side of the literature on the equity premium puzzle. The subjective discount factor is equal to 0.991, a number very close to unity to reflect that the calibration refers to a monthly decision problem. The parameters governing the dynamics of consumption and dividend growth are chosen to replicate as close as possible the first two moments and the autocorrelation functions that are observed in the data. A quick comparison of tables 8 and 9 shows that it is indeed the case that a small degree of autocorrelation cannot be rejected in the cross-section of G-7 countries studied in this paper. The correlations are chosen so that the model produce average correlations of consumption and dividend growths that are in line with the data. Indeed the model delivers a low correlation of consumption growths of about 0.25 that compares well with the average correlation of 0.13 in our cross-section of countries and that appears to be particularly accurate for the US (see table 7, top panel). This also applies to the model implied international correlation of dividend growths of 0.135 vis a vis the 0.108 of the data, for which there is not much cross-sectional variability.

3 Results

In this section we report the results obtained using the benchmark calibration and we highlight the mechanisms that allow the model in being successful in accounting for a large set of international finance anomalies. Table 9 reports the results of the model regarding the variables of interest.

3.1 International Sharpe ratios

Table 4 reports the Sharpe ratios for the countries analyzed in this paper. A large number of studies has already shown that for most countries stocks command a large risk-premium, although there appears to be some cross-sectional variability. On average the Sharpe ratios are in the order of 40%. The model is able to replicate this number, thanks to the long-run risk channel also featured in the studies by Bansal and Yaron (2004) and Colacito and Croce (2006).

Equivalently, the model produces volatile enough stochastic discount factors in the sense of Hansen and Jagannathan (1991). This is apparent, by looking at the loadings on the shocks in the home stochastic discount factor. If we abstract from stochastic volatility, the innovation to the intertemporal marginal rate of substitution can be expressed as:

$$m_{t+1} - E_t\left[m_{t+1}\right] \quad = \quad \frac{1}{\theta}\left(\frac{\delta\lambda_1\varphi_{e1}}{1-\delta\rho_1}\varepsilon_{1,t+1} + \frac{\delta\lambda_2\varphi_{e2}}{1-\delta\rho_2}\varepsilon_{2,t+1} + (1+\theta)\varepsilon_{c,t+1}\right) \tag{7}$$

Stochastic discount factors are going to be more volatile, the more agents care about the temporal distribution of risk (i.e. the lower the preference parameter θ is in absolute terms), and the more persistent the shocks to the predictive components of consumption growth (i.e. ρ_1 and ρ_2) are. A symmetric argument applies to the foreign country.

3.2 Exchange rate volatility

As reported in table 3, exchange rate movements are usually in a neighborhood of 11% in annualized terms, with the only notable exceptions of the rate of change of the US dollar vs. the Canadian dollar and of the German Marc vs. the French Franc. The model is able to match this number exactly. The insight can be found in the equation describing the growth of the real exchange rate. If we abstract from stochastic volatility and by exploiting the symmetric calibration, it is possible to write the innovations to the exchange rate growth as:

$$\Delta e_{t+1} - E_t\Delta e_{t+1} = \sigma\left(\varepsilon_{c,t+1} - \varepsilon^*_{c,t+1}\right) + \sigma\varphi_e\left[\frac{1}{\theta}\frac{\delta\left(\lambda_2-\lambda_1\right)}{1-\delta\rho}\right]\left(\varepsilon_{1,t+1} - \varepsilon_{2,t+1}\right)$$

A number of forces are at work here. The risk-sensitive (i.e. the preference parameter θ) and the long-run risk (i.e. the persistence parameter ρ) channels that are responsible for the ability of the model of matching the high international Sharpe ratios tend to increase the volatility of exchange rate fluctuations. This makes intuitive sense: the more long-lasting shocks are and the more sensitive agents are to these risks, the more they would like to use international financial markets to trade them away. This puts pressure on exchange rates to appreciate and depreciate more frequently.

Quite clearly, the international correlation of shocks plays an important role in this mechanism. If shocks were perfectly correlated across countries there would be no risk-sharing opportunity and the exchange rate would be constant over time. Our calibration features a high correlation of long-run shocks and a low correlation of short-run shocks. Our results indicate that this calibration is both consistent with the degree of international correlation of consumption growth in major industrialized countries and with the afore mentioned volatility of exchange rate movements.

3.3 Correlation of international asset returns

Table 5 reports the correlations of real excess returns and real risk-free rates for G-7 countries. These correlations range from moderately to extremely high. A consistent finding seems to be that real risk-free rates tend to be more cross-country correlated than the corresponding stock markets excess returns. The cross-sectional variability within these two sets of correlations is remarkably low, suggesting that a model that is able to replicate average correlations of 0.5 and 0.7 for excess returns and risk-free rates respectively, can be regarded as doing a good job for every single pair of countries. An important empirical remark is that the correlation of real risk-free rates appears to be different from one, although being extremely high. This finds support in a number of studies, including Mishkin (1984).

The case of no stochastic volatility is useful to illustrate the ability of the model of producing less than perfectly correlated real risk-free rates. In this case, risk-free rates are:

$$r_t = \overline{r}_t + \lambda_1 z_{1,t} + \lambda_2 z_{2,t} \qquad \text{and} \qquad r_t^* = \overline{r}_t^* + \lambda_2 z_{1,t} + \lambda_2 z_{2,t}$$

As the loadings on the predictive factors of consumption growth are different in the two countries and as long as the correlation of $z_{1,t}$ and $z_{2,t}$ is different from one, the model can deliver less than perfectly correlated real risk-free rates.

A similar argument can be made to explain the degree of correlation of the returns on the dividend claim assumed in (4). Using the Campbell and Shiller (1988) approximation

$$r_{d,t+1} \quad = \quad k_0 + \Delta d_{t+1} - k_1 dp_{t+1} + dp_t$$

where the dividend yield functions are

$$
\begin{aligned}
dp_t &= \overline{dp} + \frac{\lambda_1 - \lambda_{d_1}}{1 - k_1 \rho_1} z_{1,t} + \frac{\lambda_2 - \lambda_{d_2}}{1 - k_1 \rho_2} z_{2,t} \\
dp_t^* &= \overline{dp}^* + \frac{\lambda_2 - \lambda_{d_2}}{1 - k_1 \rho_2} z_{1,t} + \frac{\lambda_1 - \lambda_{d_1}}{1 - k_1 \rho_1} z_{2,t}
\end{aligned} \tag{8}
$$

plus a stochastic volatility term, that in figure 4 is shown not to affect our results. If dividend yields were constant, as they are in the CRRA setup without long-run risks, returns would be as correlated as their cash flows. Since dividends are almost uncorrelated across countries this would lead to the counterfactual outcome of poorly correlated returns. However, the model is able to produce highly cross-country correlated dividend yields, as confirmed by looking at the equations in (8). Hence we can expect the model to be successful in producing highly correlated international stock markets despite the fact that fundamentals appear to be mostly disconnected.

3.4 Backus and Smith anomaly

Backus and Smith (1993) pointed out that differences in cross-country consumption appear to be almost disconnected from exchange rate fluctuations. Table 2 highlights this finding by reporting the correlations between consumption growth differentials and exchange rate growths. This correlations range from about -0.2 to about 0.2 and in most cases they are extremely close to zero, leading to an average correlation of 0.072.

The Backus and Smith (1993) anomaly regarding this lack of correlation is resolved in this model. Forgoing for the moment the role of stochastic volatility (which we will later show to play a marginal role as far as this anomaly is concerned), the following system of equations sheds light into the mechanism that brings the model close to the data:

$$
\Delta e_{t+1} - E_t \Delta e_{t+1} = \sigma \left(\varepsilon_{c,t+1} - \varepsilon_{c,t+1}^* \right) + \sigma \varphi_e \left[\frac{1}{\theta} \frac{\delta(\lambda_2 - \lambda_1)}{1 - \delta \rho} \right] \left(\varepsilon_{1,t+1} - \varepsilon_{2,t+1} \right)
$$
$$
\left(\Delta c_{t+1} - \Delta c_{t+1}^* \right) - E_{t-1} \left(\Delta c_{t+1} - \Delta c_{t+1}^* \right) = \sigma \left(\varepsilon_{c,t+1} - \varepsilon_{c^*,t+1} \right)
$$

In order to lower the correlation between these two variables below one, we need to engineer a negative correlation among the last term of the exchange rate innovation and the innovation to consumption growth.

First of all, if the representative consumers in the two countries had standard time additive preferences –which is the case if $\theta \to -\infty$– exchange rates and consumption differentials would be perfectly correlated. As a concern about the timing of the resolution of uncertainty is brought into the economy, the model has the potential of breaking this correlation. Therefore the sign of the loading on $(\varepsilon_{1,t+1} - \varepsilon_{2,t+1})$ and the degree of correlation between $(\varepsilon_{1,t+1} - \varepsilon_{2,t+1})$ and $(\varepsilon_{c,t+1} - \varepsilon_{c^*,t+1})$ play key roles.

It is shown in the appendix that the signs of the coefficients β_1 and β_2 only depends on the assumption regarding the asymmetric exposure of consumption growth in the two countries to the two source of long-run risk.[6] As we mentioned above, this implies standard time-separable CRRA preferences, for which we can confirm the finding of a large body of the literature that exchange rate growth is perfectly correlated with consumption growth differentials. As we move away from the CRRA case, the role of long-run risks gets more and more important and it is possible to engineer a smaller correlation by letting ε_1 have a high degree of co-movement with ε_{c^*} and a low one with ε_c. The symmetrical argument applies to the correlations of ε_2 with ε_c and ε_{c^*}.

3.5 Uncovered Interest Parity regressions

A number of empirical studies have reported the results of the uncovered interest rate parity regression[7]

$$\log \frac{e_{t+1}}{e_t} = \alpha + \beta \left(r_t - r_t^* \right) + \xi_{t+1} \tag{9}$$

In table 1 we report the estimated slope coefficient β along with its standard error. To summarize the results we estimate only two slopes: one that summarizes all the possible UIP regressions in which the US are the home country and one which involves all countries other than the US. The resulting systems can be estimated via GMM.

[6]The only exception is the limiting case in which $\gamma \to 1$, when $\beta_1 = \beta_2 = 0$.

[7]For consistency with the rest of the analysis, the regression is ran with real instead of nominal variables. Refer to Sercu and Uppal (2000) for a review of the literature on real and nominal UIP.

The findings confirm that the null hypothesis of a regression coefficient equal to one can be rejected at any conventional level of significance. The estimated β should be approximately equal to zero, although it may be a desirable feature of the model to produce a negative regression coefficient as well.

Stochastic volatility plays the crucial role in enabling the model to explain the smaller-than-one coefficient that we typically find when running UIP-like regressions. The model counterpart of regression (9) is

$$m_{t+1}^{*} - m_{t+1} \quad = \quad \alpha + \beta \left(-\log E_t \exp \left\{ m_{t+1} \right\} + \log E_t \exp \left\{ m_{t+1}^{*} \right\} \right) + \xi_{t+1}$$

which, exploiting log-normality of the stochastic discount factors, can be written as

$$m_{t+1}^{*} - m_{t+1} \quad = \quad \alpha + \beta \left(E_t m_{t+1}^{*} - E_t m_{t+1} + \frac{1}{2} Var_t m_{t+1}^{*} - \frac{1}{2} Var_t m_{t+1} \right) + \xi_{t+1}$$

The estimated slope of the regression is simply

$$\widehat{\beta} \quad = \quad \frac{Var(p_t) + \frac{1}{2} Cov(p_t, q_t)}{Var(p_t + \frac{1}{2} q_t)} \tag{10}$$

where $p_t = E_t m_{t+1}^{*} - E_t m_{t+1}$ and $q_t = Var_t m_{t+1}^{*} - Var_t m_{t+1}$. It is apparent from (10) that $\widehat{\beta}$ is smaller than one only if the model features stochastic volatility and p_t co-varies negatively with q_t. This point was raised in the seminal work by Fama (1984) and more recently by Backus et al. (2001) in the context of affine term structure models.

In our model, these two quantities are equal to

$$\begin{aligned} p_t &= \underbrace{(\lambda_1 - \lambda_1^{*})}_{>0} z_{1,t} + \underbrace{(\lambda_2 - \lambda_2^{*})}_{<0} z_{2,t} + \underbrace{\frac{(v_1 R v_1' - v_1^{*} R v_1^{*\prime})}{2\theta^2}}_{>0} \lambda_t + \underbrace{\frac{(v_2 R v_2' - v_2^{*} R v_2^{*\prime})}{2\theta^2}}_{<0} \lambda_t^{*} \\ q_t &= \underbrace{\frac{(v_1^{*} R v_1^{*\prime} - \widetilde{v}_1 R \widetilde{v}_1')}{2\theta^2}}_{<0} \lambda_t + \underbrace{\frac{(\widetilde{v}_2^{*} R \widetilde{v}_2^{*\prime} - v_2 R v_2')}{2\theta^2}}_{>0} \lambda_t^{*} \end{aligned}$$

Given the signs of the coefficients, it is possible to satisfy the negative covariance requirement simply by setting to zero the correlations between λ's and between λ'a and z's. The question becomes whether we can obtain enough negative correlation to lower the slope of the UIP regression to the levels that we obtain in the data.

We analyze this question in the next section, but we can anticipate that the answer depends crucially on the persistence of the low frequency components of consumption growth and on the coefficient of risk-aversion γ.

4 Discussion

In this section we perform a number of sensitivity exercises that enable us to account for the contribution of each ingredient of the model to the explanation of the six international anomalies.

4.1 The case for risk-sensitive preferences

Figure 1 shows the performance of the model when the baseline calibration is employed with the only exception of the coefficient γ varying between 1 and 10. When the two representative consumers have CRRA preference (i.e. when $\gamma = 1$), the model fails along several dimensions, as time additive preferences essentially shut down the contribution of long-run risks. This means that the stochastic discount factors will display and extremely low volatility, which automatically leads the model to fail in accounting for the large equity premium and for the degree of volatility of exchange rate growth. The correlation between consumption differential and exchange rate is almost equal to one (the small departure is due to the presence of stochastic volatility) as their dynamics are both driven by idiosyncratic shocks to consumption only. There is almost no role for stochastic volatility when $\gamma = 1$, as shown in the last panel of the figure. The two facts that the model is able to explain independently of the coefficient of risk aversion are the correlations of risk-free rates and of excess returns. As we will document in figures 3 and 2, it is the persistence and the cross-country correlation of long-run risks that plays a major role relative to these two moments.

As γ increases, the model gets closer to the data. It should not strike as a surprise that a higher risk aversion can account for the equity premium that we observe in G-7 countries. But it is important to stress that this can be achieved for values of γ that are significantly lower than what is commonly found in the equity premium puzzle literature. As risk aversion increases, the consumption agents' consumption smoothing desire increases as well. This prompts the exchange rate to adjust more in response to shock of any nature taking place in any country. Hence the ability of the model to account for the degree of volatility of changes in the currency value of about 11% in annualized terms.

As higher risk aversion increases the sensitivity of exchange rates to all shocks, the model's performance regarding the Backus and Smith anomaly is also improved. In this case a key role is played by our assumptions about the structure of the correlation matrix of idiosyncratic and permanent shocks to consumption. According to it, when the home country experiences a long-run shock to consumption, the foreign country is likely to be affected by a short run shock of equal sign. The lagged response of consumption to long-run shocks, implies a temporary higher consumption in the home country. The exchange rate is likely to adjust by virtue of two offsetting forces. One the one hand home country's consumers would like to smooth over time the benefits of the shortly-lived shock and, on the other hand, foreign country's consumers would like to change their consumption profile today in anticipation of the better growth prospects. Hence depending on the actual realizations of the shocks, exchange rates could move either way, making them uncorrelated with the shift in consumption differentials whose sign is unambiguous.

4.2 The case for long-run risks

Figure 2 shows the performance of the model as the persistence of the predictive components of consumption growth increases. International data impose very strong identifying restrictions on what this persistence should be. With the only possible exception of the correlation of risk-free rates, it is only for very high values of $\rho_1 = \rho_2$ that the model is able to account for the moments of interest. For the small predictive components of consumption growth to play a role in international asset pricing, it must be the case that they decay at a very slow rate.

4.3 The case for two factors

The system of equations in section 3.3 documents that, aside for a small stochastic volatility effect, the presence of only one common long-run component would lead risk-free rates to be perfectly correlated across countries, a counterfactual outcome. We think that it is a major advantage of our model to produce short rates that are significantly different from one another. Furthermore, with only a common predictive component in the dynamics of consumption growths, Colacito and Croce (2006) show that exchange rates movements are entirely driven by the dynamics of idiosyncratic shocks to consumption. This implies that exchange rates and consumption differentials should be perfectly correlated, that is the Backus and Smith (1993) anomaly would emerge back in our model.

Hence, we believe that the assumption of two predictive factors is an important one and that it enriches the set of international moments that can be accounted for by this class of models. In figure 3 we document that a high correlation of the two consumption factors is still needed.

4.4 The case for stochastic volatility

Figure 4 shows that the introduction of stochastic volatility is important only for explaining the forward premium anomaly. A different way of interpreting this picture is that a small amount of time varying economic uncertainty does not prevent the model from being able to explain the other five puzzles on the table. Stochastic volatility alone would not be able to produce a significantly smaller than unity regression coefficient, as illustrated in the previous three figures.

5 Concluding remarks

In this paper, we presented a calibrated model that is able to explain a large of international finance anomalies. The crucial ingredients are utility functions that display a preference for the timing of the resolution of the uncertainty, slowly moving predictive factors of consumption and dividend growths, and a particular structure of the correlation matrix of international consumption growth at different frequencies. Combined with a small amount of stochastic volatility, the model is also able to account for the forward premium anomaly.

A number of interesting questions arise in the context of this class of models. What are the general equilibrium conditions that give rise to the consumption dynamics postulated in the long-run risks literature? Can we use the larger set of international data to get better estimates of the otherwise hard to detect predictive components of consumption? Can we assess the benefits of international financial diversification in the context of this model that can explain the joint dynamics of international quantities and prices? These are just of the questions that future developments of this line of research should address.

Appendix A

The setup of the model is reported in section 2. The solution consists of three steps:

1. I approximate $\lambda_t^{1/2}$, $\lambda_t^{*1/2}$ and h_t starting from λ_t and λ_t^*
2. I solve for the two value functions
3. I obtain the stochastic discount factors

Useful approximations

A first order linear approximation $\lambda_t^{1/2}$, $\lambda_t^{*1/2}$ and $h_t = \lambda_t^{1/2}\lambda_t^{*1/2}$ around the steady state $\overline{\lambda} = \overline{\lambda^*} = \sigma^2$ delivers:

$$\begin{aligned}
\lambda_t^{1/2} &\approx \sigma\left(1-\rho_\lambda\right) + \rho_\lambda \lambda_{t-1}^{1/2} + \frac{1}{2}\varphi_\lambda \varepsilon_{\lambda,t} \\
\lambda_t^{*1/2} &\approx \sigma\left(1-\rho_\lambda\right) + \rho_\lambda \lambda_{t-1}^{*1/2} + \frac{1}{2}\varphi_\lambda \varepsilon_{\lambda,t}^* \\
h_t &\approx \sigma^2\left(1-\rho_\lambda\right)^2 + \sigma\left(1-\rho_\lambda\right)\rho_\lambda\left(\lambda_{t-1}^{1/2} + \lambda_{t-1}^{*1/2}\right) + \rho_\lambda^2 h_{t-1} + \\
&\quad + \frac{\varphi_\lambda}{2}\left[\sigma\left(1-\rho_\lambda\right) + \rho_\lambda \lambda_{t-1}^{*1/2}\right]\varepsilon_{\lambda,t} + \frac{\varphi_\lambda}{2}\left[\sigma\left(1-\rho_\lambda\right) + \rho_\lambda \lambda_{t-1}^{1/2}\right]\varepsilon_{\lambda,t}^*
\end{aligned}$$

Solving for the value functions

I shall start by expressing the utility functions in a more convenient form. Subtracting $\log C_t$ from both sides of the home country utility function reported in (1), it is possible to obtain:

$$\begin{aligned}
V_t &= U_t - \log C_t \\
&= \theta\delta \log E_t \exp\left\{\frac{U_{t+1} - \log C_{t+1} + \log C_{t+1} - \log C_t}{\theta}\right\} \\
&= \theta\delta \log E_t \exp\left\{\frac{V_{t+1} + \Delta c_{t+1}}{\theta}\right\}
\end{aligned}$$

Similarly for the foreign country:

$$V_t^* = \theta\delta \log E_t \exp\left\{\frac{V_{t+1}^* + \Delta c_{t+1}^*}{\theta}\right\}$$

Guess that the solution of V_t and V_t^* is linear:

$$V_t = A + B_1 z_{1,t} + B_2 z_{2,t} + D_\lambda \lambda_t + D_{\lambda^*}\lambda_t^* + D_h h_t + E_\lambda \lambda_t^{1/2} + E_{\lambda^*}\lambda_t^{*^{1/2}}$$

with the solution for V_t^* taking on the same expression, but with the coefficients being indexed by a $(^*)$. It follows that:

$$\begin{aligned} V_t = {} & \theta\delta \log E_t \exp\left\{\frac{1}{\theta}\left[\mu_c + \lambda_1 z_{1,t} + \lambda_2 z_{2,t} + \lambda_t^{1/2}\varepsilon_{c,t+1} + A + \right.\right. \\ & + B_1\left(\rho_1 z_{1,t} + \varphi_{e_1}\lambda_t^{1/2}\varepsilon_{1,t+1}\right) + B_2\left(\rho_2 z_{2,t} + \varphi_{e_2}\lambda_t^{1/2}\varepsilon_{2,t+1}\right) + \\ & + D_\lambda\left(\sigma^2(1-\rho_\lambda) + \rho_\lambda\lambda_t + \varphi_\lambda\lambda_t^{1/2}\varepsilon_{\lambda,t+1}\right) + D_{\lambda^*}\left(\sigma^2(1-\rho_\lambda) + \rho_\lambda\lambda_t^* + \varphi_\lambda\lambda_t^{*^{1/2}}\varepsilon_{\lambda,t+1}^*\right) + \\ & + D_h\left(\sigma^2(1-\rho_\lambda)^2 + \sigma(1-\rho_\lambda)\rho_\lambda\left(\lambda_t^{1/2} + \lambda_t^{*^{1/2}}\right) + \rho_\lambda^2 h_t + \right. \\ & \left. + \frac{\varphi_\lambda}{2}\left[\sigma(1-\rho_\lambda) + \rho_\lambda\lambda_t^{*^{1/2}}\right]\varepsilon_{\lambda,t+1} + \frac{\varphi_\lambda}{2}\left[\sigma(1-\rho_\lambda) + \rho_\lambda\lambda_t^{1/2}\right]\varepsilon_{\lambda,t+1}^*\right) + \\ & \left.\left. + E_\lambda\left(\sigma(1-\rho_\lambda) + \rho_\lambda\lambda_t^{1/2} + \frac{1}{2}\varphi_\lambda\varepsilon_{\lambda,t+1}\right) + E_{\lambda^*}\left(\sigma(1-\rho_\lambda) + \rho_\lambda\lambda_t^{*^{1/2}} + \frac{1}{2}\varphi_\lambda\varepsilon_{\lambda,t+1}^*\right)\right]\right\} \end{aligned}$$

Defining the two vectors v_1 and v_2 as

$$\begin{aligned} v_1 &= \begin{bmatrix} B_1\varphi_{e_1}, & 0, & 1, & 0, & D_\lambda\varphi_\lambda, & D_h\frac{\varphi_\lambda}{2}\rho_\lambda \end{bmatrix} \\ v_2 &= \begin{bmatrix} 0, & B_2\varphi_{e_2}, & 0, & 0, & D_h\frac{\varphi_\lambda}{2}\rho_\lambda, & D_{\lambda^*}\varphi_\lambda \end{bmatrix} \end{aligned}$$

the value function V_t can be rewritten as:

$$\begin{aligned} V_t = {} & A + \delta(\lambda_1 + B_1\rho_1) z_{1,t} + \delta(\lambda_2 + B_2\rho_2) z_{2,t} + \delta\left(D_\lambda\rho_\lambda + \frac{v_1 R v_1'}{2\theta}\right)\lambda_t + \\ & + \delta\left(D_{\lambda^*}\rho_\lambda + \frac{v_2 R v_2'}{2\theta}\right)\lambda_t^* + \delta\left(D_h\rho_\lambda^2 + \frac{v_1 R v_2'}{2\theta}\right)h_t + \delta\left[D_h\sigma\rho_\lambda(1-\rho_\lambda) + E_\lambda\rho_\lambda\right]\lambda_t^{1/2} + \\ & + \delta\left[D_h\sigma\rho_\lambda(1-\rho_\lambda) + E_{\lambda^*}\rho_\lambda\right]\lambda_t^{*^{1/2}} \end{aligned}$$

The solution for the parameter A is easy to find, but I will omit it, because it is not going to affect the stochastic discount factors. All other parameters can be found by matching coefficients:

$$B_1 = \frac{\delta \lambda_1}{1 - \delta \rho_1}, \qquad B_2 = \frac{\delta \lambda_2}{1 - \delta \rho_2}$$

The three parameters D_λ, D_{λ^*} and D_h are the solution of a second order system:

$$D_\lambda = \delta \left[D_\lambda \rho_\lambda + \frac{v_1 R v_1'}{2\theta} \right], D_{\lambda^*} = \delta \left[D_{\lambda^*} \rho_\lambda + \frac{v_2 R v_2'}{2\theta} \right], D_h = \delta \left[D_h \rho_\lambda^2 + \frac{v_1 R v_2'}{2\theta} \right]$$

Out of the 8 possible solutions, I shall select the one with largest coefficients, because that would be the one that maximizes utility, given that λ_t, λ_t^* and h_t are always greater than zero by definition. The last two parameters are

$$E_\lambda = \frac{\delta D_h \sigma \rho_\lambda (1 - \rho_\lambda)}{1 - \delta \rho_\lambda}, \qquad E_{\lambda^*} = \frac{\delta D_h \sigma \rho_\lambda (1 - \rho_\lambda)}{1 - \delta \rho_\lambda}$$

For the foreign country, the procedure is identical with two exceptions:

1. variables indexed by a $(^*)$ will be used
2. the vectors v_1 and v_2 are replaced by

$$\begin{aligned} v_1^* &= \begin{bmatrix} B_1^* \varphi_{e_1}, & 0, & 0, & 0, & D_\lambda \varphi_\lambda, & D_h^* \frac{\varphi_\lambda}{2} \rho_\lambda \end{bmatrix} \\ v_2^* &= \begin{bmatrix} 0, & B_2^* \varphi_{e_2}, & 0, & 1, & D_h^* \frac{\varphi_\lambda}{2} \rho_\lambda, & D_{\lambda^*} \varphi_\lambda \end{bmatrix} \end{aligned}$$

respectively.

Solving for the stochastic discount factors

Having solve for the two value functions, it is now possible to solve for the stochastic discount factors. I shall start with the home country:

$$\begin{aligned} m_{t+1} &= \log\delta - \Delta c_{t+1} + \log \frac{\exp\left\{\frac{V_{t+1}+\Delta c_{t+1}}{\theta}\right\}}{E_t \exp\left\{\frac{V_{t+1}+\Delta c_{t+1}}{\theta}\right\}} \\ &= \log\delta - \mu_c - \lambda_1 z_{1,t} - \lambda_2 z_{2,t} - \lambda_t^{1/2}\varepsilon_{c,t+1} + \frac{1}{\theta}\left(v_1\lambda_t^{1/2} + v_2\lambda_t^{*1/2}\right)\eta_{t+1} + \\ &\quad -\frac{1}{2\theta^2}\left(v_1 R v_1'\lambda_t + v_2 R v_2'\lambda_t^* + v_1 R v_2' h_t\right) \end{aligned}$$

Define $v_{0,m}$, $\widetilde{v}_1$ and v_m as:

$$\begin{aligned} v_{0,m} &= \mu_c - \log\delta \\ \widetilde{v}_1 &= v_1 + \begin{bmatrix} 0 & 0 & \theta & 0 & 0 & 0 \end{bmatrix} \\ v_m &= \begin{bmatrix} \lambda_1 & \lambda_2 & \frac{v_1 R v_1'}{2\theta^2} & \frac{v_2 R v_2'}{2\theta^2} & \frac{v_1 R v_2'}{2\theta^2} \end{bmatrix} \end{aligned}$$

and rewrite the stochastic discount factor in compact vector notation:

$$m_{t+1} = -v_{0,m} - v_m s_t' + \frac{1}{\theta}\left(\widetilde{v}_1\lambda_t^{1/2} + v_2\lambda_t^{*1/2}\right)\eta_{t+1}' \tag{11}$$

Similarly for the foreign country:

$$m_{t+1}^* = -v_{0,m} - v_m^* s_t' + \frac{1}{\theta}\left(v_1^*\lambda_t^{1/2} + \widetilde{v}_2^*\lambda_t^{*1/2}\right)\eta_{t+1}' \tag{12}$$

where

$$\begin{aligned} \widetilde{v}_2^* &= v_2^* + \begin{bmatrix} 0 & 0 & 0 & \theta & 0 & 0 \end{bmatrix} \\ v_m^* &= \begin{bmatrix} \lambda_1^* & \lambda_2^* & \frac{v_1^* R v_1^{*\prime}}{2\theta^2} & \frac{v_2^* R v_2^{*\prime}}{2\theta^2} & \frac{v_1^* R v_2^{*\prime}}{2\theta^2} \end{bmatrix} \end{aligned}$$

Appendix B

Derivation of returns

In this section we show show how to derive the risk free rates, the real exchange rate and the returns on assets that pay the dividend whose process is specified in 4.

Risk-free rates

Given the processes for the stochastic discount factors in equations (11) and (12), risk free rates can be computed as:

$$
\begin{aligned}
r_t &= -\log E_t \exp\{m_{t+1}\} \\
&= v_{0,m} + v_m s_t' - \frac{1}{2\theta^2}\left(\widetilde{v}_1 R \widetilde{v}_1' \lambda_t + v_2 R v_2 \lambda_t^* + \widetilde{v}_1 R v_2' h_t\right) \\
&= v_{0,m} + \widetilde{v}_m s_t'
\end{aligned} \tag{13}
$$

where

$$
\widetilde{v}_m = \left[\begin{array}{ccccc} \lambda_1, & \lambda_2, & \frac{v_1 R v_1' - \widetilde{v}_1 R \widetilde{v}_1'}{2\theta^2}, & 0, & \frac{(v_1 - \widetilde{v}_1) R v_2'}{2\theta^2} \end{array}\right]
$$

Similarly for the foreign country:

$$
r_t^* = v_{0,m} + \widetilde{v}_m^* s_t' \tag{14}
$$

where

$$
\widetilde{v}_m^* = \left[\begin{array}{ccccc} \lambda_1^*, & \lambda_2^*, & 0, & \frac{v_2^* R v_2^{*\prime} - \widetilde{v}_2^* R \widetilde{v}_2^{*\prime}}{2\theta^2}, & \frac{v_1^* R \left(v_2^* - \widetilde{v}_2^*\right)}{2\theta^2} \end{array}\right]
$$

Real exchange rate

Assuming that markets are complete, the growth of the real exchange rate must equal the difference between the two stochastic discount factors:

$$\log\frac{e_{t+1}}{e_t} = m^*_{t+1} - m_{t+1}$$
$$= (v_m - v^*_m)\, s'_t + \frac{1}{\theta}\left[(v^*_1 - \widetilde{v}_1)\,\lambda_t^{1/2} + (\widetilde{v}^*_2 - v_2)\,\lambda_t^{*1/2}\right]\eta'_{t+1} \quad (15)$$

Asset returns

Campbell and Shiller (1988) show that the logarithm of the return of an asset can be approximated as:

$$r_{d,t+1} \approx k_0 + \Delta d_{t+1} - k_1 dp_{t+1} + dp_t$$

where Δd_{t+1} is growth of the dividend paid by the asset, dp_{t+1} is the logarithm of the dividend to price ratio and k_0 and k_1 are approximation constants defined as

$$k_0 = \log\left(1 + \exp\left\{\overline{dp}\right\}\right) - \frac{\exp\left\{\overline{dp}\right\}\overline{dp}}{1 + \exp\left\{\overline{dp}\right\}}, \qquad k_1 = \frac{1}{1 + \exp\left\{\overline{dp}\right\}}$$

with $\overline{dp}$ being the steady state of the dividend to price ratio about which the approximation is taken.

I shall solve for the logarithm of the dividend to price ratio. Guess that the solution of dp_t is linear in $z_{1,t}$, $z_{2,t}$, λ_t, λ^*_t, h_t, $\lambda_t^{1/2}$, $\lambda_t^{*1/2}$:

$$dp_t = v_{0,p} + v_{1,p}z_{1,t} + v_{2,p}z_{2,t} + v_{\lambda,p}\lambda_t + v_{\lambda^*,p}\lambda^*_t + v_{h,p}h_t + v_{e,p}\lambda_t^{1/2} + v_{e^*,p}\lambda_t^{*1/2}$$

and use the Campbell and Shiller (1988) approximation of returns in the Euler equation

$$E_t\left[\exp\{m_{t+1}\}\exp\{r_{t+1}\}\right] = 1$$

to write

$$E_t \quad \Big[\exp\Big\{-v_{0,m} - v_m s'_t + \frac{1}{\theta}\left(\widetilde{v}_1\lambda_t^{1/2} + v_2\lambda_t^{*1/2}\right)\eta'_{t+1} +$$
$$k_0 + \mu_d + \lambda_{d_1}z_{1,t} + \lambda_{d_2}z_{2,t} + \varphi_d\lambda_t^{1/2}\varepsilon_{d,t+1} - k_1 dp_{t+1} + dp_t\Big\}\Big] = 1$$

Substitute the guess solution in the Euler equation:

$$
\begin{aligned}
-dp_t \quad = \quad & \log E_t \exp\Big\{-k_1 v_{0,p} - k_1 v_{1,p}\left(\rho_1 z_{1,t} + \varphi_{e_1}\lambda_t^{1/2}\varepsilon_{1,t+1}\right) - k_1 v_{2,p}\left(\rho_2 z_{2,t} + \varphi_{e_2}\lambda_t^{1/2}\varepsilon_{2,t+1}\right) \\
& -k_1 v_{\lambda,p}\left(\sigma^2(1-\rho_\lambda) + \rho_\lambda\lambda_t + \varphi_\lambda\lambda_t^{1/2}\varepsilon_{\lambda,t+1}\right) - k_1 v_{\lambda^*,p}\left(\sigma^2(1-\rho_\lambda) + \rho_\lambda\lambda_t^*\right. \\
& \left.+\varphi_\lambda\lambda_t^{*^{1/2}}\varepsilon_{\lambda,t+1}^*\right) - k_1 v_{h,p}\left(\sigma^2(1-\rho_\lambda)^2 + \sigma(1-\rho_\lambda)\rho_\lambda\left(\lambda_t^{1/2} + \lambda_t^{*^{1/2}}\right) + \rho_\lambda^2 h_t + \right. \\
& \left.+\frac{\varphi_\lambda}{2}\left[\sigma(1-\rho_\lambda) + \rho_\lambda\lambda_t^{*^{1/2}}\right]\varepsilon_{\lambda,t+1} + \frac{\varphi_\lambda}{2}\left[\sigma(1-\rho_\lambda) + \rho_\lambda\lambda_t^{1/2}\right]\varepsilon_{\lambda,t+1}^*\right) + \\
& -k_1 v_{e,p}\left(\sigma(1-\rho_\lambda) + \rho_\lambda\lambda_t^{1/2} + \frac{1}{2}\varphi_\lambda\varepsilon_{\lambda,t+1}\right) - k_1 v_{e^*,p}\left(\sigma(1-\rho_\lambda) + \rho_\lambda\lambda_t^{*^{1/2}}\right. \\
& \left.\left.+\frac{1}{2}\varphi_\lambda\varepsilon_{\lambda,t+1}^*\right) + k_0 + \mu_d - v_{0,m} - v_m s_t' + \frac{1}{\theta}\left(\widetilde{v}_1\lambda_t^{1/2} + v_2\lambda_t^{*^{1/2}}\right)\eta_{t+1}'\right]\Big\}
\end{aligned}
$$

For convenience, denote

$$
\begin{aligned}
v_m &= \begin{bmatrix} v_{1,m}, & v_{2,m}, & v_{\lambda,m}, & v_{\lambda^*,m}, & v_{h,m} \end{bmatrix} \\
w_{ap} &= \begin{bmatrix} 0, & 0, & 0, & 0, & \frac{\varphi_\lambda}{2}\left[1+\sigma(1-\rho_\lambda)\right], & \frac{\varphi_\lambda}{2}\left[1+\sigma(1-\rho_\lambda)\right], & 0, & 0 \end{bmatrix} \\
b_1 &= -k_1\begin{bmatrix} v_{1,p}\varphi_e, & 0, & 0, & 0, & v_{\lambda,p}\varphi_\lambda, & v_{h,p}\frac{\varphi_\lambda\rho_\lambda}{2}, & -\frac{\varphi_d}{k_1}, & 0 \end{bmatrix} + \frac{1}{\theta}\begin{bmatrix} \widetilde{v}_1, & 0, & 0 \end{bmatrix} \\
b_2 &= -k_1\begin{bmatrix} 0, & v_{2,p}\varphi_e, & 0, & 0, & v_{h,p}\frac{\varphi_\lambda\rho_\lambda}{2}, & v_{\lambda^*,p}\varphi_\lambda, & 0, & 0 \end{bmatrix} + \frac{1}{\theta}\begin{bmatrix} v_2, & 0, & 0 \end{bmatrix}
\end{aligned}
$$

The solution of the dividend to price ratio schedule can be obtained by matching coefficients. The easiest to compute are the ones that pre-multiply the low frequency components of consumption growth:

$$
v_{1,p} = \frac{v_{1,m} - \lambda_{d_1}}{1 - k_1\rho_1}, \qquad v_{2,p} = \frac{v_{2,m} - \lambda_{d_2}}{1 - k_1\rho_2}
$$

The coefficients $v_{\lambda,p}$, $v_{\lambda^*,p}$ and $v_{h,p}$ are the solution of a second order system:

$$
\begin{aligned}
v_{\lambda,p} &= k_1 v_{\lambda,p}\rho_\lambda + v_{\lambda,m} - b_1\widetilde{R}b_1', \qquad v_{\lambda^*,p} = k_1 v_{\lambda^*,p}\rho_\lambda + v_{\lambda^*,m} - b_2\widetilde{R}b_2' \\
v_{h,p} &= k_1 v_{h,p}\rho_\lambda^2 + v_{h,m} - b_1\widetilde{R}b_2'
\end{aligned}
$$

Given the calibration that is used in the paper, the solution of this system turns out to be unique. The remainder of the coefficients is equal to:

$$
\begin{aligned}
v_{e,p} \quad &= \quad \frac{k_1 v_{h,p}\sigma(1-\rho_\lambda)\rho_\lambda}{1-k_1\rho_\lambda}, \quad v_{e^*,p} = v_{e,p} \\
v_{0,p} \quad &= \quad \frac{1}{1-k_1}\Big\{k_1\left[\sigma^2(1-\rho_\lambda)(v_{\lambda,p} + v_{\lambda^*,p}) + \sigma^2(1-\rho_\lambda)^2 v_{h,p} + \sigma(1-\rho_\lambda)(v_{e,p} + v_{e^*,p})\right] + \\
& \quad -w_{ap}\widetilde{R}w_{ap}' + v_{0,m} - k_0 + \mu_d\Big\}
\end{aligned}
$$

For the foreign country, the procedure is identical with two exceptions:

1. variables indexed by a $(^{*})$ will be used
2. the vectors v_1 and v_2 are replaced by

$$\begin{aligned} b_1^* &= -k_1 \left[\begin{array}{cccccccc} v_{1,p}^* \varphi_e, & 0, & 0, & 0, & v_{\lambda,p}^* \varphi_\lambda, & v_{h,p}^* \frac{\varphi_\lambda \rho_\lambda}{2}, & 0, & 0 \end{array} \right] + \frac{1}{\theta} \left[\begin{array}{ccc} v_1^*, & 0, & 0 \end{array} \right] \\ b_2^* &= -k_1 \left[\begin{array}{cccccccc} 0, & v_{2,p}^* \varphi_e, & 0, & 0, & v_{h,p}^* \frac{\varphi_\lambda \rho_\lambda}{2}, & v_{\lambda^*,p}^* \varphi_\lambda, & 0, & -\frac{\varphi_d}{k_1} \end{array} \right] + \frac{1}{\theta} \left[\begin{array}{ccc} \widetilde{v}_2^*, & 0, & 0 \end{array} \right] \end{aligned}$$

respectively.

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Table 1: GMM estimation with all predictive variables

	α		β		J-statistic	
US	0.016	(0.078)	-0.136	(0.295)	9.272	(0.507)
Rest of the world	0.024	(0.003)	0.130	(0.178)	51.455	(0.016)

Notes - Uncovered Interest Rate Parity regressions. The table reports the estimated paramters for the UIP regressions

$$\Delta e_{t+1} \quad = \quad \alpha + \beta\left(r_t - r_t^*\right)$$

The row labeled 'US' show the results of the UIP regressions pooled for the case in which the US are the home country. The row labeled 'Rest of the World' shows the pooled estimates for all other countries. The columns next to the estimated parameters report standard errors. The last column shows the p-values associated to the J-statistic.

Table 2: The Backus and Smith anomaly

	US	CAN	JPN	GER	UK	FRA	ITA	Average
US	-	-0.223	-0.013	0.166	-0.146	-0.007	-0.037	-0.043
CAN	-0.223	-	0.098	0.185	0.007	0.105	0.153	0.054
JPN	-0.013	0.098	-	0.220	-0.053	0.251	0.069	0.095
GER	0.166	0.185	0.220	-	0.108	0.123	0.157	0.160
UK	-0.146	0.007	-0.053	0.108	-	0.042	0.224	0.030
FRA	-0.007	0.105	0.251	0.123	0.042	-	0.085	0.100
ITA	-0.037	0.153	0.069	0.157	0.224	0.085	-	0.108
Average								0.072

"Notes - Correlations of real consumption growth differentials and exchange rate growths. Data are qurterly from 1974:1 to 1998:4. The country on the row is home, and the country on the column is foreign."

Table 3: Volatilities of exchange rate growth

	US	CAN	JPN	UK	GER	FRA	ITA	Average
US	-	4.506	13.131	10.965	12.479	11.629	11.620	10.722
CAN	4.506	-	13.615	11.435	13.127	12.422	12.365	12.593
JPN	13.131	13.615	-	12.585	11.597	11.470	13.038	12.461
UK	10.965	11.435	12.585	-	9.784	8.967	9.540	10.462
GER	12.479	13.127	11.597	9.784	-	4.631	8.804	9.588
FRA	11.629	12.422	11.470	8.967	4.631	-	7.003	8.898
ITA	11.620	12.365	13.038	9.540	8.804	7.003	-	10.150
Average								10.696

Notes - Volatily of real exchange rate growths. Figures are annualized from quarterly. The sample is 1974:1 to 1998:4.

Table 4: Sharpe Ratios

	US	CAN	JPN	GER	UK	FRA	ITA	Average
Sharpe ratio	58.694	31.278	22.038	43.169	46.286	40.904	22.099	37.781

Notes - Annualized Sharpe ratios computed from quarterly returns from 1974:1 to 1998:12.

Table 5: International Correlations of Asset Returns

	US	CAN	JPN	GER	UK	FRA	ITA	Average
US	-	0.821	0.481	0.487	0.637	0.588	0.433	0.574
CAN	0.821	-	0.392	0.428	0.540	0.486	0.407	0.512
JPN	0.481	0.392	-	0.294	0.422	0.359	0.389	0.389
GER	0.487	0.428	0.294	-	0.478	0.630	0.498	0.469
UK	0.637	0.540	0.422	0.478	-	0.541	0.376	0.499
FRA	0.588	0.486	0.359	0.630	0.541	-	0.587	0.532
ITA	0.433	0.407	0.389	0.498	0.376	0.587	-	0.448
Average								0.489

	US	CAN	JPN	GER	UK	FRA	ITA	Average
US	-	0.695	0.664	0.502	0.709	0.520	0.625	0.619
CAN	0.695	-	0.758	0.686	0.754	0.719	0.671	0.714
JPN	0.664	0.758	-	0.609	0.659	0.648	0.784	0.687
GER	0.502	0.686	0.609	-	0.630	0.701	0.563	0.615
UK	0.709	0.754	0.659	0.630	-	0.761	0.737	0.708
FRA	0.520	0.719	0.648	0.701	0.761	-	0.837	0.698
ITA	0.625	0.671	0.784	0.563	0.737	0.837	-	0.703
Average								0.678

Notes - The top panel reports the correlations of international stock markets' excess returns. The bottom panel shows the correlations of international risk-free rates. Data are quarterly over the sample 1974:1 to 1998:4.

Table 6: Baseline Calibration

Risk aversion	γ	8.000
Subjective discount factor	δ	0.991
Average consumption growth	μ_c	0.001
Average dividend growth	μ_d	0.001
Loading on $z_{1,t}$ in Δc_{t+1}	λ_1	5.500
Loading on $z_{2,t}$ in Δc_{t+1}	λ_2	1.000
Loading on $z_{1,t}$ in Δc^*_{t+1}	λ^*_1	1.000
Loading on $z_{2,t}$ in Δc^*_{t+1}	λ^*_2	5.500
Loading on $z_{1,t}$ in Δc_{t+1}	λ_{d_1}	24.000
Loading on $z_{2,t}$ in Δc_{t+1}	λ_{d_2}	2.000
Loading on $z_{1,t}$ in Δc^*_{t+1}	$\lambda^*_{d_1}$	2.000
Loading on $z_{2,t}$ in Δc^*_{t+1}	$\lambda^*_{d_2}$	24.000
Persistence of first predictive factor	ρ_1	0.987
Persistence of second predictive factor	ρ_2	0.987
Persistence of stochastic volatility	ρ_λ	0.950
Predictive factors to consumption growth standard error ratio	φ_e	8.80E-03
Dividend to consumption growth standard error ratio	φ_d	4.500
Stochastic volatility to consumption growth std error ratio	φ_λ	3.50E-04
Standard error of the shock to consumption growth	σ	0.006

Correlations

Predictive factors	$\rho_{1,2}$	0.600
Predictive factors and consumption growth	$\rho_{1,c} = \rho_{2,c^*}$	0.060
Predictive factors and consumption growth	$\rho_{1,c^*} = \rho_{2,c}$	0.600
Predictive factors and stochastic volatility	$\rho_{1,\lambda} = \rho_{2,\lambda^*} = -\rho_{2,\lambda} = -\rho_{1,\lambda^*}$	-0.285
Consumption growth and stochastic volatility	$\rho_{c,\lambda} = \rho_{c^*,\lambda^*} = -\rho_{c,\lambda^*} = -\rho_{c^*,\lambda}$	0.470
Dividend growths	ρ_{d,d^*}	0.050
Consumption growths	ρ_{c,c^*}	0.100

Notes - The calibration is for the model simulated at a monthly frequency. The two countries are assumed to have the same calibration, with the only exception of the loadings on the predictive factors of consumption growths. The bottom part of the table shows the correlations of the shocks. Any correlation that is not reported in the table is calibrated to zero.

Table 7: Correlations of quantities

	US	CAN	JPN	GER	UK	FRA	ITA	Average
US	-	0.356	0.077	0.242	0.214	0.266	-0.018	0.228
CAN	0.356	-	-0.074	0.094	0.259	0.165	0.099	0.150
JPN	0.077	-0.074	-	0.067	0.054	0.151	0.126	0.067
GER	0.242	0.094	0.067	-	0.115	-0.066	-0.068	0.064
UK	0.214	0.259	0.054	0.115	-	0.311	-0.001	0.158
FRA	0.266	0.165	0.151	-0.066	0.311	-	0.134	0.160
ITA	-0.018	0.099	0.126	-0.068	-0.001	0.134	-	0.045
Average								0.125

	US	CAN	JPN	GER	UK	FRA	ITA	Average
US	-	0.151	0.014	-0.020	-0.053	0.019	0.045	0.026
CAN	0.151	-	0.057	0.173	0.255	-0.035	-0.110	0.082
JPN	0.014	0.057	-	-0.002	0.076	0.062	0.239	0.074
GER	-0.020	0.173	-0.002	-	0.203	0.202	0.183	0.123
UK	-0.053	0.255	0.076	0.203	-	0.219	0.244	0.157
FRA	0.019	-0.035	0.062	0.202	0.219	-	0.343	0.135
ITA	0.045	-0.110	0.239	0.183	0.244	0.343	-	0.157
Average								0.108

Notes - The top panel reports the correlations of real international consumption growths. The bottom panel shows the correlations of real international dividend growths. Data are quarterly over the sample 1974:1 to 1998:4.

Table 8: Consumption and dividend growth in the cross-section

	US	CAN	JPN	ITA	GER	FRA	UK	Average
Std. dev.	1.639	1.840	1.962	2.054	1.976	1.765	2.325	1.937
AC(1)	0.334	0.102	0.240	0.619	-0.007	0.006	0.124	0.203
AC(2)	0.349	0.160	0.292	0.329	0.028	-0.003	0.183	0.191
AC(3)	0.195	0.225	0.386	0.019	0.163	-0.034	0.149	0.158
AC(4)	0.100	0.008	0.176	-0.144	0.029	-0.007	-0.061	0.014
AC(5)	0.087	0.102	0.233	-0.220	0.031	-0.038	0.140	0.048
AC(6)	0.077	0.061	0.236	-0.264	-0.062	-0.056	-0.017	-0.004
AC(7)	-0.018	0.105	0.172	-0.223	0.073	0.004	0.098	0.030
AC(8)	-0.127	0.019	0.094	-0.104	-0.088	-0.009	-0.049	-0.038
AC(9)	-0.073	0.033	0.159	0.000	-0.051	-0.001	-0.058	0.001
AC(10)	-0.076	0.143	0.280	-0.004	0.022	-0.032	-0.044	0.041
Std.dev.	16.226	5.087	4.172	14.350	10.262	8.228	6.976	9.329
AC(1)	-0.493	0.592	0.347	0.197	0.019	0.104	0.341	0.158
AC(2)	0.275	0.456	0.000	-0.039	-0.033	0.017	-0.025	0.093
AC(3)	-0.189	0.372	0.111	0.157	0.113	0.024	0.017	0.086
AC(4)	0.234	0.206	0.257	0.197	0.101	0.257	0.019	0.182
AC(5)	-0.162	0.143	-0.006	0.076	0.077	0.091	-0.079	0.020
AC(6)	0.189	-0.032	-0.042	-0.006	-0.064	0.028	-0.104	-0.004
AC(7)	-0.253	-0.143	0.153	0.148	0.016	0.070	0.087	0.011
AC(8)	0.288	-0.138	0.247	0.304	-0.079	0.420	0.053	0.156
AC(9)	-0.311	-0.238	0.056	-0.036	0.008	-0.129	-0.030	-0.097
AC(10)	0.209	-0.287	-0.090	-0.014	-0.003	-0.070	-0.156	-0.059

Notes - The top panel shows standard deviations and auto-correlation functions of consumption growth in G-7 countries. The bottom panel reports the same moments for the distribution of dividend growth.

Table 9: Results with baseline calibration

Volatility of exchange rate growth	10.929	(9.105,12.954)
UIP regression slope	-0.062	(-0.609,0.458)
Sharpe ratio	40.564	(2.783,76.447)
Backus and Smith anomaly	-0.056	(-0.279,0.165)
Correlation of risk-free rates	0.793	(0.189,0.954)
Correlation of excess returns	0.422	(0.232,0.578)
Volatility of consumption growth	1.909	(1.547,2.388)
AC(1)	0.347	(0.081,0.598)
AC(2)	0.158	(-0.154,0.484)
AC(3)	0.14	(-0.161,0.473)
AC(4)	0.125	(-0.141,0.448)
Correlation of consumption growths	0.247	(-0.032,0.546)
Volatility of dividend growth	9.845	(8.115,12.045)
AC(1)	0.109	(-0.142,0.384)
AC(2)	0.088	(-0.161,0.376)
AC(3)	0.087	(-0.161,0.358)
AC(4)	0.075	(-0.164,0.356)
Correlation of dividend growths	0.135	(-0.113,0.380)

Notes - The first six rows of the table report the median and the 95% confidence interval for the volatility of exchange rate growth, the slope of the uncovered interest rate parity regression, the Sharpe ratio, the correlation between consumption growth differentials and exchange rate growth (Backus and Smith anomaly), the correlation of risk-free rates, and the correlation of excess returns. The following lines report medians and 95% confidence intervals for various consumption and dividend growth moments. The results are obtained by simulating 1000 independent samples of size 24 years at a monthly frequency.

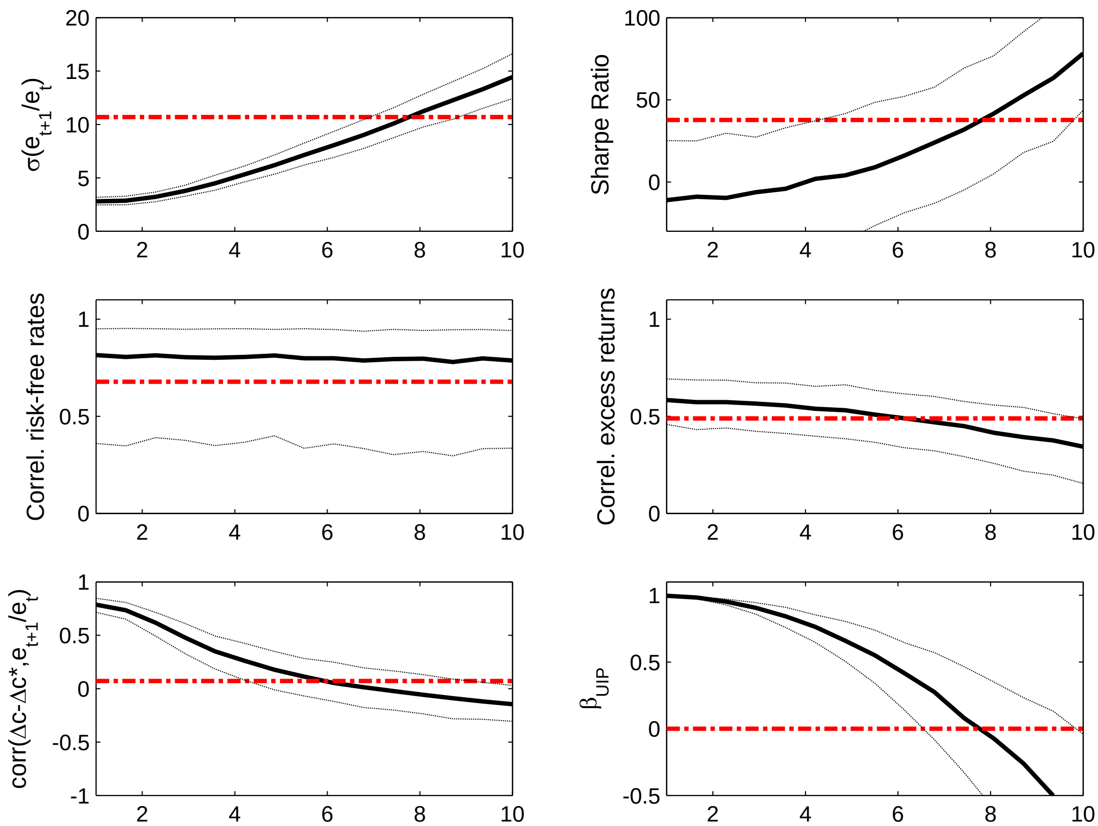

Figure 1: Long run risks model letting γ vary. The six subplots depict the behavior (from the top left to the right bottom) of the volatility of exchange rate growth, the Sharpe ratio, correlation of risk-free rates, the correlation of excess returns, the correlation of consumption growth differentials and exchange rate growth and of the slope in the uncovered interest rate parity regression. For each panel, the solid line represents the median across 1000 simulations of length 24 years sampled at a monthly frequency, while the dotted lines are 95% confidence intervals and the dashed dotted line is the data measured equivalent.

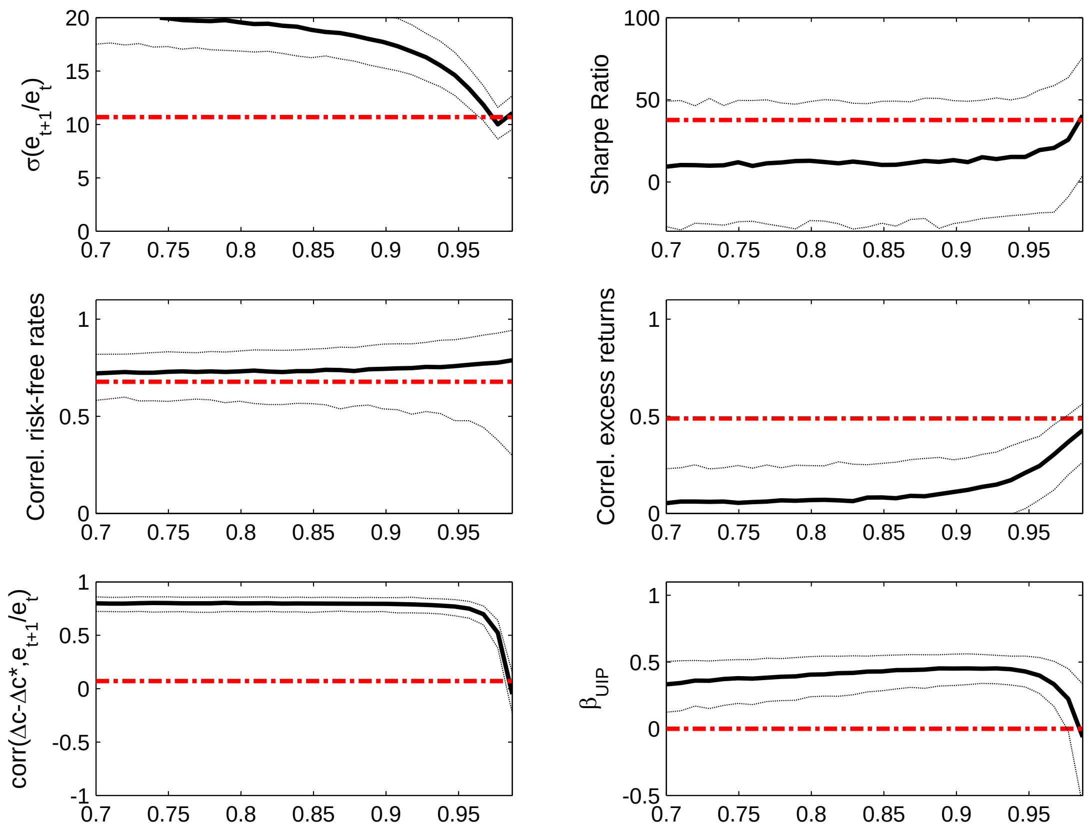

Figure 2: Long run risks model letting $\rho_1 = \rho_2$ vary. The six subplots depict the behavior (from the top left to the right bottom) of the volatility of exchange rate growth, the Sharpe ratio, correlation of risk-free rates, the correlation of excess returns, the correlation of consumption growth differentials and exchange rate growth and of the slope in the uncovered interest rate parity regression. For each panel, the solid line represents the median across 1000 simulations of length 24 years sampled at a monthly frequency, while the dotted lines are 95% confidence intervals and the dashed dotted line is the data measured equivalent.

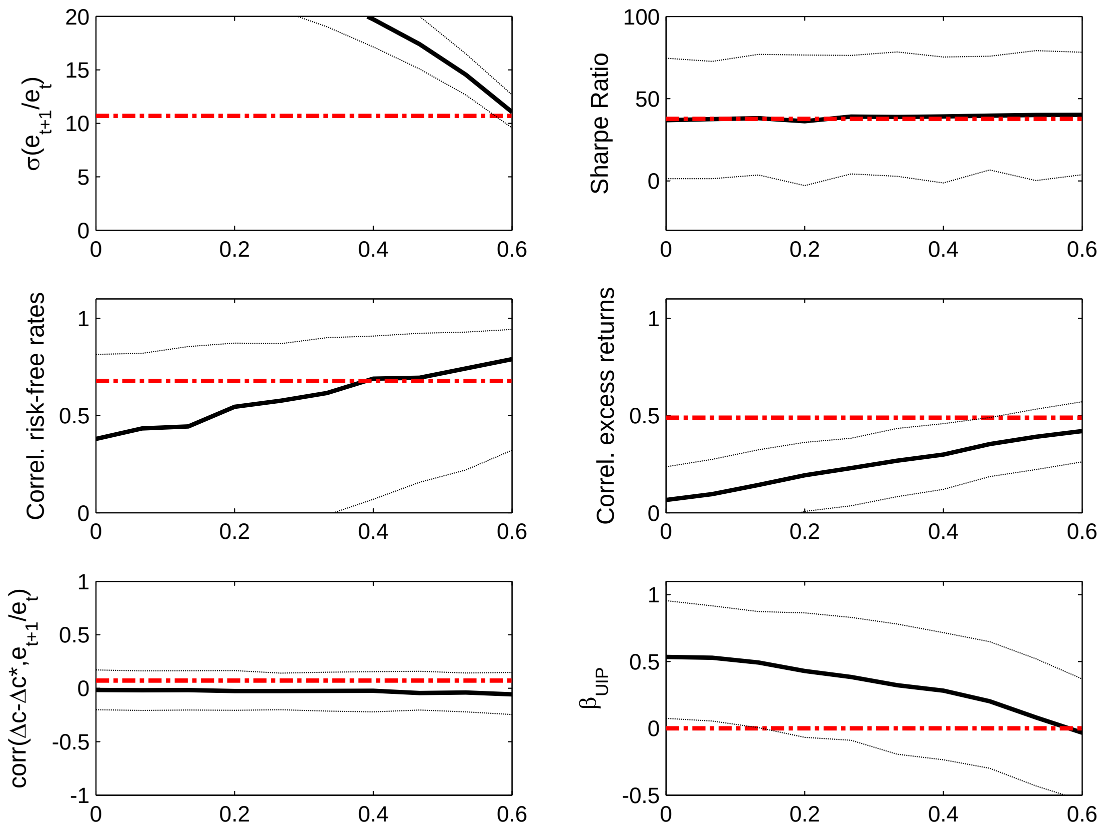

Figure 3: Long run risks model letting $\rho_{1,2}$ vary. The six subplots depict the behavior (from the top left to the right bottom) of the volatility of exchange rate growth, the Sharpe ratio, correlation of risk-free rates, the correlation of excess returns, the correlation of consumption growth differentials and exchange rate growth and of the slope in the uncovered interest rate parity regression. For each panel, the solid line represents the median across 1000 simulations of length 24 years sampled at a monthly frequency, while the dotted lines are 95% confidence intervals and the dashed dotted line is the data measured equivalent.

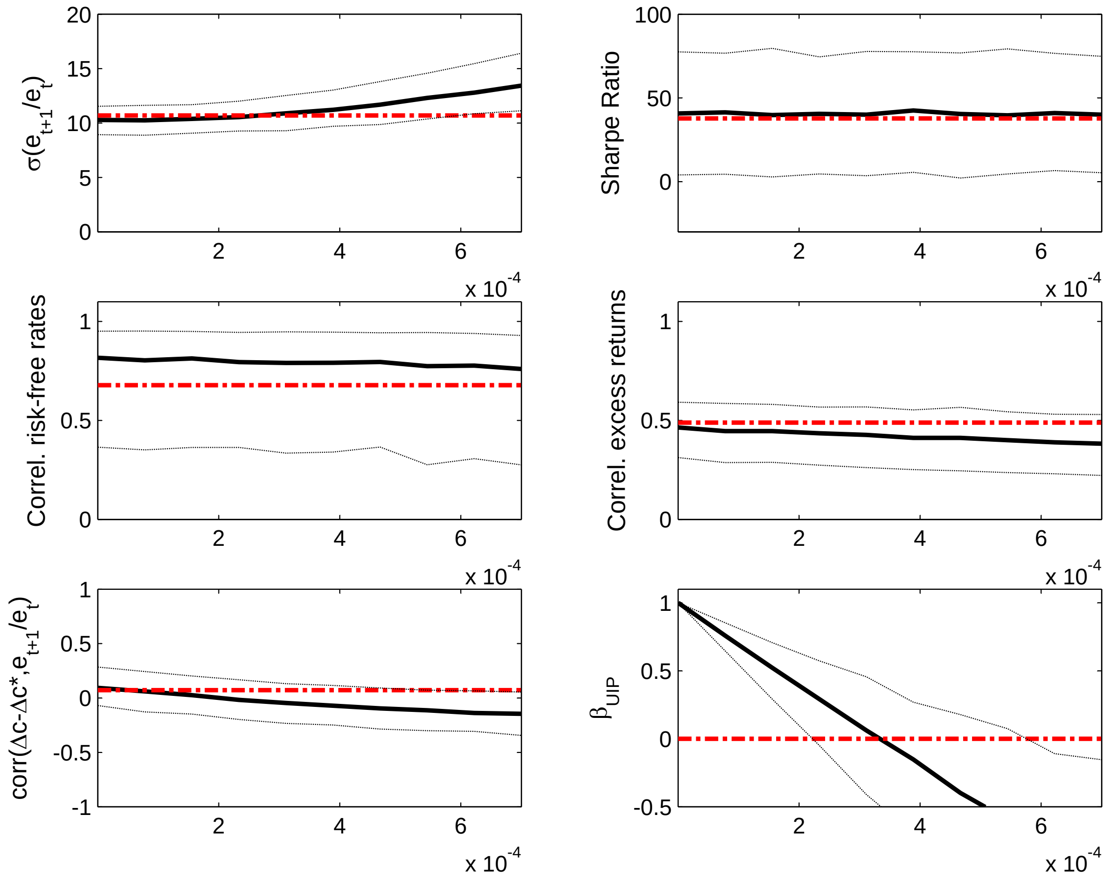

Figure 4: Long run risks model letting φ_λ vary. The six subplots depict the behavior (from the top left to the right bottom) of the volatility of exchange rate growth, the Sharpe ratio, correlation of risk-free rates, the correlation of excess returns, the correlation of consumption growth differentials and exchange rate growth and of the slope in the uncovered interest rate parity regression. For each panel, the solid line represents the median across 1000 simulations of length 24 years sampled at a monthly frequency, while the dotted lines are 95% confidence intervals and the dashed dotted line is the data measured equivalent.