Disagreement vs. Uncertainty: Investment Dynamics and Business Cycles

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The paper

- **Goal**: analyze the connection between
  - time-varying volatility in productivity
  - the cross-section of firms' investments
  - the dynamics of business cycles' fluctuations
- A very ambitious undertaking!
- Main ingredients:
  1. time-varying vol in firm level productivity
  2. time-varying precision in the signal about aggregate productivity
  3. convex adjustment costs
- The key novelty lies in the interaction of 1+2
  $\rightarrow$ Focus of my discussion

Provide direct evidence of productivity process?

- Estimate the state-space system

$$\begin{bmatrix} z_{i,t} \\ s_t \end{bmatrix} = A \cdot \begin{bmatrix} z_{i,t-1} \\ x_t \end{bmatrix} + B_t \cdot \begin{bmatrix} e_{i,t} \\ \xi_t \end{bmatrix}$$

$$x_t = C \cdot x_{t-1} + D \cdot v_t$$

where

  - $z_{i,t}$ is firm specific productivity (observable)
  - x_t is aggregate productivity (**un**observable)
  - s_t is signal about aggregate productivity (observable)
    $\rightarrow$ Use macro/financial variables as proxies
    (e.g. stock returns, p/d ratios, analysts' forecasts,...)

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- Currently: parameters are either borrowed from literature or based on indirect evidence (eg firms' disagreement)

Why is this relevant?

- Tight identification of parameters is key for model's performance
- Two illustrative examples
  1. Relative persistence of aggregate versus firm specific productivity
     $\rightarrow$ The calibration assumes $\rho_x < \rho_z$ based on literature
  2. Correlation between signal precision and idiosyncratic productivity uncertainty
     $\rightarrow$ The model assumes orthogonal vol shocks

Relative persistence

- Simulate x and z according to

$$[\textit{Homoskedastic}] \quad x_t = \rho_x \cdot x_{t-1} + 0.15 \cdot \mathbf{v_t}$$

$$[\textit{Heteroskedastic}] \quad z_{i,t} = \rho_z \cdot z_{i,t-1} + \sigma_{e,t-1} \cdot \mathbf{v_t}$$

$$\sigma_{e,t} = (1 - 0.93) \cdot 0.15 + 0.93 \cdot \sigma_{e,t-1} + \eta_{\sigma_e} \cdot \varepsilon_{\sigma_e,t}$$

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- Set $\rho_z = 0.97$ and $\rho_x = \{0.87, 0.88, ..., 0.97\}$
- Simulate 500 samples of size 120 observations
- Estimate $\hat{\rho}_x$ and $\hat{\rho}_z$ separately using OLS
- A laboratory to study how the presence of TVV affects our ability to test that $\rho_x \geq \rho_z$

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Time-varying vol: set $\rho_z = 0.97$ and $\eta_{\sigma_e} = 0.007$

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ρ_x	0.87	0.89	0.91	0.93	0.95	0.97
$H_0 : \rho_x \geq \rho_z$	0.02	0.06	0.13	0.24	0.36	0.51

Relative persistence: Take-aways

- The presence of time-varying volatility hampers the tight identification of the AR coefficients
- In the paper $\rho_x = 0.93$ and $\rho_z = 0.97$: is this spread consistent with the estimation of a model with time-varying vol?
- The sign and magnitude of $\rho_z - \rho_x$ is crucial for the qualitative and quantitative performance of the model
- Providing direct evidence is relatively easy and would significantly strengthen the plausibility of the key mechanism of the model.

Correlated volatilities?

Proposition 1. Larger imprecision of signals, larger cross-sectional dispersion of beliefs about future aggregate productivity.

$$\frac{\partial \sigma_{E(x_2)}}{\partial \sigma_\xi} = \rho_x \cdot \frac{1/\sigma_e}{1/\sigma_v^2 + 1/\sigma_e^2 + 1/\sigma_\xi^2} > 0$$

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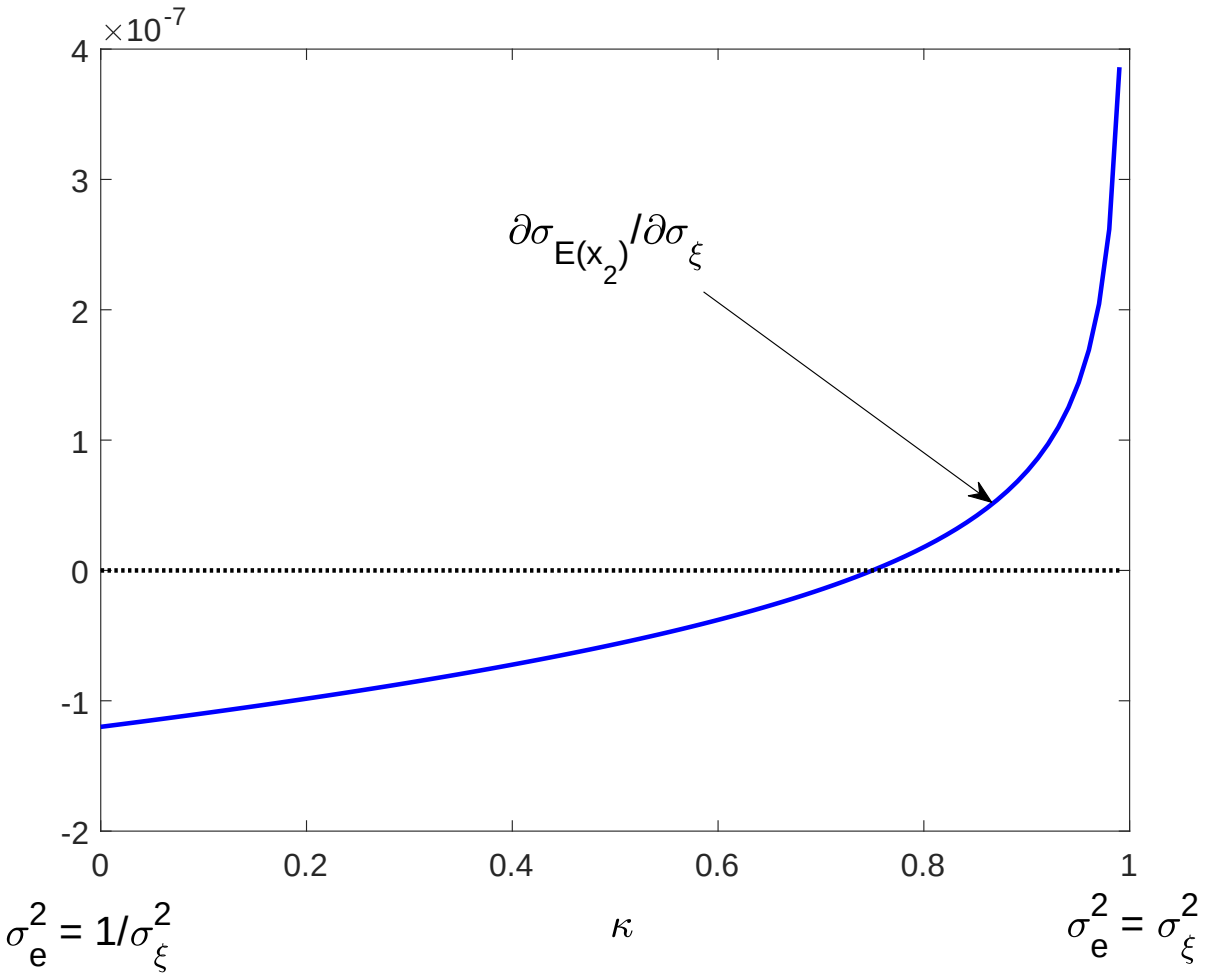
What if σ_ξ and σ_e are related?

$$\sigma_e^2 = \kappa \cdot \sigma_\xi^2 + (1-\kappa) \cdot \frac{1}{\sigma_\xi^2}$$

- $\kappa = 1$: signal's imprecision and firm uncertainty are perfectly correlated
- $\kappa = 0$: firm uncertainty is high when the signal is very precise

Signal precision and disagreement



Correlated volatilities: Take-aways

- The correlation of volatility shocks may affect the positive relationship between signal imprecision and disagreement.
- Most likely: volatilities are positively correlated and argument will hold under more general conditions.
- Corollary point:
  - in the model time-varying idiosyncratic vol is the same for all firms (tractability)
  - could less than perfectly correlated idiosyncratic vols be a source of disagreement about aggregate productivity?

Miscellanea

- Is it possible to further simplify the finite horizon model?
  $\rightarrow$ Maybe use a "GARCH-in-mean" type of specification as a reduced form way to model the outcome of the signal extraction problem.
- Can the analysis be extended to the case of time-varying vol also in aggregate productivity?
- What are the implications for the cross-section of stock returns?

Conclusion

- A very interesting and ambitious paper!
- Tightening the connection between assumption about productivity and data would significantly strengthen the paper