International Macro-Finance & Long-Run Risks: an overview

Ric *Colacito*

THE UNIVERSITY
of NORTH CAROLINA
at CHAPEL HILL

The FX volatility puzzle

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- Hard to reconcile:
  ↬ Large SDF volatility
  ↬ Low correlation of consumption
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- Reading list:
  ↬ Bansal and Yaron, JF 2004
  ↬ Brandt, Cochrane, and Santa Clara, JME 2006
  ↬ Colacito and Croce, JPE 2011

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Setup of the economy

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- Epstein, Zin and Weil preferences:

$$U_t^i = \left\{(1-\delta)(C_t^i)^{\frac{1-\gamma}{\theta}} + \delta\left[E_t(U_{t+1}^i)^{1-\gamma}\right]^{\frac{1}{\theta}}\right\}^{\frac{\theta}{1-\gamma}}, \forall i \in \{h, f\}$$

where

$$\theta = \frac{1-\gamma}{1-1/\psi}.$$

Dynamics of consumption

Based on Bansal and Yaron (JF, 2004):

$$\Delta c_t^i = \mu_c + x_{t-1}^i + \sigma \underbrace{\varepsilon_{c,t}^i}_{\text{short-run shocks}}$$

$$x_t^i = \rho x_{t-1}^i + \sigma \varphi_e \underbrace{\varepsilon_{x,t}^i}_{\text{long-run shocks}}, \qquad \forall i \in \{h, f\}$$

<u>Shocks correlations</u>:

- orthogonal within each country
- correlated across countries
  - $\looparrowright$ $\rho_c = corr(\varepsilon_{c,t}^h, \varepsilon_{c,t}^f)$
  - $\looparrowright$ $\rho_x = corr(\varepsilon_{x,t}^h, \varepsilon_{x,t}^f)$

Stochastic discount factors ($\psi = 1$)

- The stochastic discount factors are

$$m^i_{t+1} = \log\delta - \Delta c^i_{t+1} + \log \frac{\exp\{(1-\gamma)U^i_{t+1}\}}{E_t\left[\exp\{(1-\gamma)U^i_{t+1}\}\right]}$$

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- Combine:
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  - ↬ High long-run int'l correlation (U^i_{t+1})
- Economic intuition:
  - ↬ short-run: slow diffusion of technologies and ideas across countries
  - ↬ long-run: countries converge to the same technological frontier

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Exchange rates are functions of relative supplies of the two goods

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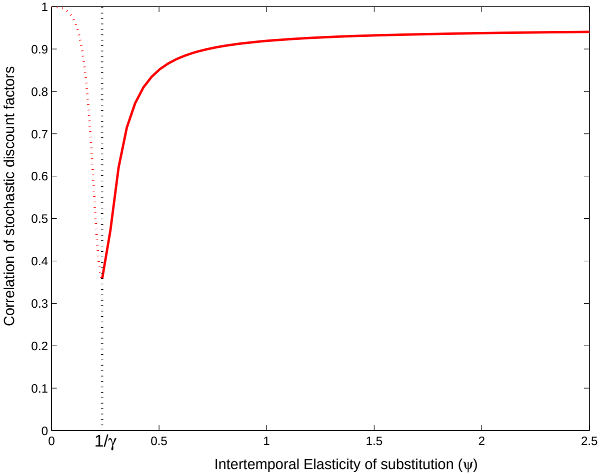
- Agents are extremely sensitive to long-run news
- Long-run risks should be very correlated to replicate FX volatility

SDF Correlation

$$m^i_{t+1} = \theta \log \delta - \frac{1}{\psi} x^i_t - \gamma\sigma\varepsilon^i_{c,t+1} + \frac{\delta(1-\gamma\psi)}{\psi(1-\rho\delta)} \sigma\varphi_e \varepsilon^i_{x,t+1}$$



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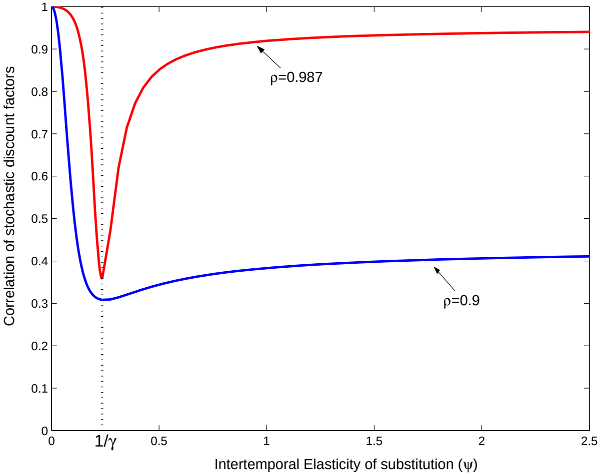

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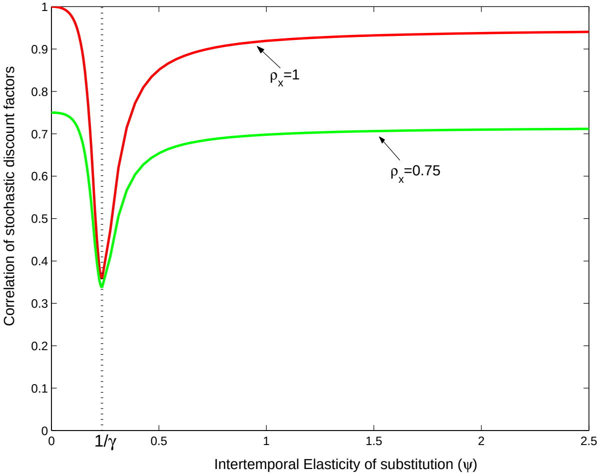

FX Volatility

$$Var\left(\frac{e_{t+1}}{e_t}\right) = \frac{2(1-\rho_x)}{\psi^2}\left\{\frac{1}{1-\rho^2} + \left[\frac{\delta(1-\gamma\psi)}{(1-\rho\delta)}\right]^2\right\}\varphi_e^2\sigma^2 + 2\gamma^2(1-\rho_c)\sigma^2$$

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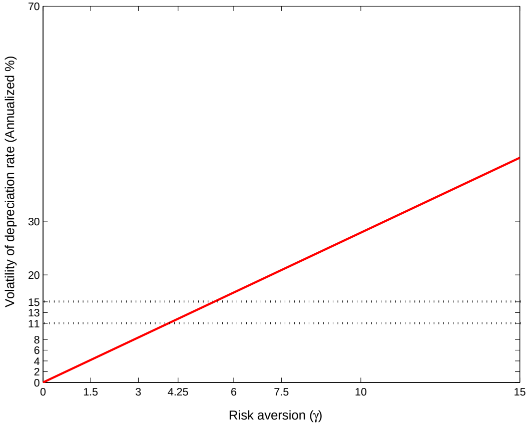

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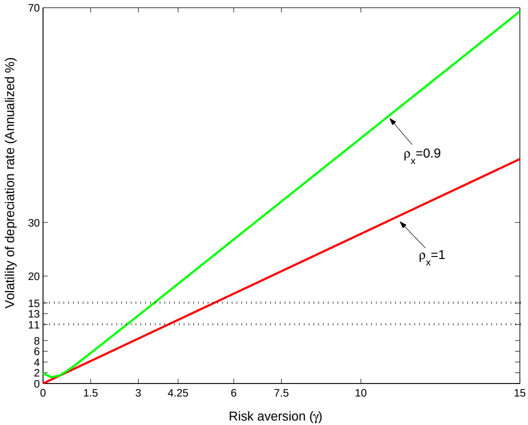

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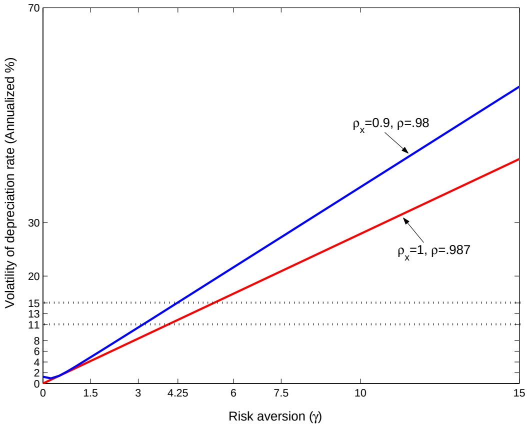

Estimation

Use predictive regressions

$$\Delta c_t^i = \mu_c + \underbrace{Z_{t-1}^i \cdot \beta^i}_{x_{t-1}^i} + \varepsilon_{c,t}^i$$

where $Z_t^i = \left[pd_t^i, r_{f,t}^i, \Delta c_t^i, \Delta cy_t^i, default_t^i\right], \forall i \in \{h, f\}$.

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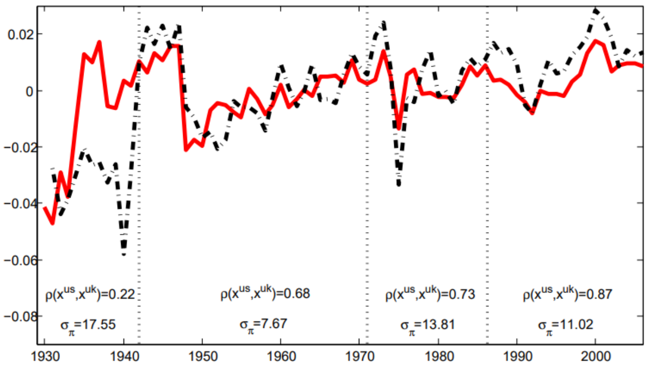
	$F-stat$		R^2		ρ_x		$corr\left(x^{US}, x^{UK}\right)$
	US	UK	US	UK	US	UK	
Pd and risk-free	6.641	3.476	0.145	0.086	0.768	0.787	0.579
	(0.002)	(0.036)			(0.076)	(0.074)	[0.137, 0.896]
All predictive variables	6.585	6.852	0.315	0.278	0.672	0.759	0.758
	(0.000)	(0.000)			(0.074)	(0.074)	[0.531, 0.922]
Pd only	7.299	6.013	0.086	0.076	0.885	0.726	0.849
	(0.008)	(0.016)			(0.065)	(0.099)	[0.762, 0.941]
VAR	11.719	4.204	0.325	0.149	0.869	0.765	0.717
	(0.000)	(0.008)			(0.058)	(0.075)	[0.172, 0.953]

FX vol and correlation of predictive components



FX tend to be smoother when predictive components are more correlated

Estimating preference parameters

	Conditional Estimation			Joint Estimation		
	P/D	P/D, R_f	All	P/D	P/D, R_f	All
ψ	4.094	1.371	1.276	2.781	2.719	2.001
	[0.398, 6.739]	[0.391, 1.792]	[0.404, 1.992]	[0.243, 2.971]	[0.288, 3.030]	[0.274, 3.099]
γ	4.359	3.402	2.936	3.387	3.225	4.147
	[3.181, 9.575]	[2.282, 7.376]	[2.144, 6.339]	[1.014, 10.267]	[1.090, 9.509]	[1.167, 10.802]
ρ_x	-	-	-	0.997	0.997	0.988
				[0.427, 0.999]	[0.413, 0.999]	[0.662, 0.999]
$\rho\left(x^{US}, x^{UK}\right)$	-	-	-	0.879	0.867	0.830
				[0.618, 0.936]	[0.582, 0.939]	[0.471, 0.963]
Wald-stat p-value	-	-	-	0.000	0.000	0.000
$\gamma - 1/\psi$ p-value	0.000	0.002	0.008	0.000	0.001	0.000
J-stat p-value	0.000	0.000	0.000	0.035	0.000	0.011

- GMM estimation with Euler equation restrictions and first two moments of FX
- Low risk aversion
- IES around 2